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Efficient BER Approximation for Amplify-and-Forward Cooperative Wireless Relay System with BPSK in Rayleigh Fading Channels

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Abstract. In this paper, we derive an efficient and simple closed-form approximate formula of bit error rate (BER) for amplify-and-forward (AF) cooperative wireless system in Rayleigh fading channels. In contrast to the conventional integral-form expression, the proposed simple formula can be utilized to efficiently investigate and provide an explicit insight into the performance behavior of the AF cooperative relay system.

Keywords: amplify-and-forward (AF), cooperative wireless system, Rayleigh fading channels

1 Introduction

In recent years, a cooperative wireless relay technique has been extensively studied as an effective method to combat fading, provide diversity gain, and enhance the transmission reliability over conventional wireless systems in fading environments. Specifically, various cooperative transmission strategies have been proposed [1-4], including amplify-and-forward (AF), decode-and-forward (DF), etc. It was shown in [1] that both high data transmission rate and link reliability can be achieved through the cooperative relay schemes by employing an additional relay. In [2] and [3], the error rate performance of dual-hop single-link communications has been analyzed without the relay diversity. For various types of cooperative transmission strategies, such as coherent/noncoherent AF and DF, the comprehensive performance analysis was presented in [4], where Rayleigh fading channels were considered for all links. Thus, starting from the research results given in [4], the purpose of this paper is to derive a simple but efficient closed-form approximate formula of bit error rate (BER) especially for a coherent AF cooperative relay system in Rayleigh fading channels.

2 System Model and BER Performance Evaluation

In this paper, a wireless cooperative relay system is considered, which consists of a source (S), relay (R), and destination (D) nodes, as illustrated in Figure 1. Due to the

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space limit, we focus on the performance assessment . Thus, we refer the interested reader to [4] for more detailed cooperation protocol and combining at the destination.

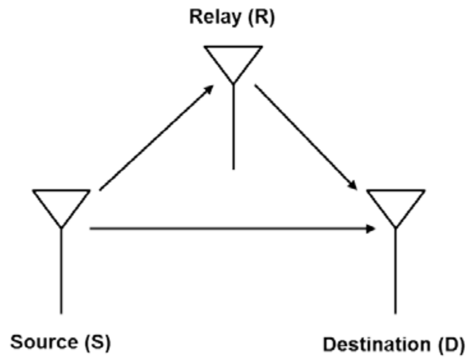

Fig. 1. Cooperative wireless relay system

From the combined SNR of the maximal ratio combining (MRC) at D and by exploiting the MGF-based method [5], the average BER for binary phase shift keying (BPSK) can be expressed as [4]

$$\begin{aligned}\overline{P}_b &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \mathcal{MGF}_{\gamma_{\text{Overall}}}\left(-\frac{1}{\sin^2\phi}\right)d\phi \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \mathcal{MGF}_{\gamma_{S,D}}\left(-\frac{1}{\sin^2\phi}\right)\mathcal{MGF}_{\gamma_{S,R,D}}\left(-\frac{1}{\sin^2\phi}\right)d\phi\end{aligned} \tag{1}$$

with

$$\mathcal{MGF}_{\gamma_{S,D}}(s) = \frac{1}{1+\rho_{S,D}s} \text{ and } \mathcal{MGF}_{\gamma_{S,R,D}}(s) \approx {}_2F_1\left(1,2;\frac{3}{2};-\frac{\rho_{S,R}}{4}s\right), \tag{2}$$

where $\rho_{i,j}$ denotes the average SNR of *i-j* link with $(i,j)\in\{(S,R),(S,D),(R,D)\}$, ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the Gauss hypergeometric function [6], and the approximation of $\mathcal{MGF}_{\gamma_{S,R,D}}(s)$ is given in [3]. It was indicated in [4] that Eq. (1) can be only computed by numerical integration technique.

However, by judiciously utilizing the approximation technique presented in [7], Eq. (1) can be further simplified in the following closed-form expression as

$$\begin{aligned}\overline{P}_b &\approx \frac{1}{12}\mathcal{MGF}_{\gamma_{\text{Overall}}}(1) + \frac{1}{4}\mathcal{MGF}_{\gamma_{\text{Overall}}}\left(\frac{4}{3}\right) \\ &= \frac{1}{12}\left(\frac{1}{1+\rho_{S,D}}\right){}_2F_1\left(1,2;\frac{3}{2};-\frac{\rho_{S,R}}{4}\right) + \frac{1}{4}\left(\frac{1}{1+\frac{4}{3}\rho_{S,D}}\right){}_2F_1\left(1,2;\frac{3}{2};-\frac{\rho_{S,R}}{3}\right).\end{aligned} \tag{3}$$

Furthermore, by applying the high-SNR approximation (i.e., $\rho_{i,j} \to \infty$) and the series expansion technique as ${}_2F_1(1,2;3/2;-z) \approx 1/2z$, we can further simplify Eq. (3) and thus obtain the asymptotic BER expression as

$$\begin{aligned}\bar{P}_b^{\infty} &\approx \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(\frac{1}{\rho_{S,D}\sin^{-2}\theta}\right)\left(\frac{2}{\rho_{R,D}\sin^{-2}\theta}\right)d\theta \\ &= \frac{3}{8}\frac{1}{\rho_{S,D}\rho_{R,D}}\end{aligned}, \tag{4}$$

where it is apparently demonstrated that the coherent AF cooperative system in Rayleigh fading channels can achieve the asymptotic diversity order of 2, which agrees with the result presented in [4].

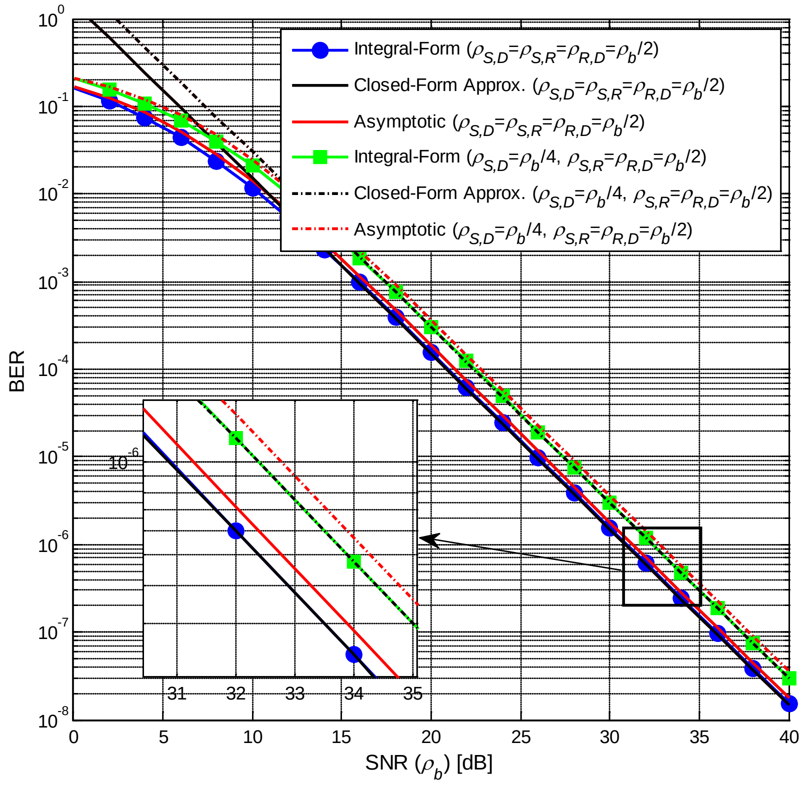

Fig. 2. Average BER versus SNR for coherent AF cooperative relay system with BPSK in Rayleigh fading channels

3 Numerical Results

We provide numerical results to show the BER performance of coherent AF cooperative relay system employing BPSK in Rayleigh fading channels, depicted in Fig. 1. In addition, for a variety of system configurations, we also consider two scenarios. That is, for a symmetric scenario, we set $\rho_{S,D} = \rho_{S,R} = \rho_{R,D} = 0.5\rho_b$, while for an asymmetric scenario, $\rho_{S,D} = 0.25\rho_b$ and $\rho_{S,R} = \rho_{R,D} = 0.5\rho_b$.

As shown in Fig. 2, the SER curves obtained from the proposed closed-form approximation in Eq. (3) are close to those from the conventional integral-form expression given in [4] for wide SNR ranges, although a small SNR gap of about 0.3dB is observed, which is due to the intrinsic approximation technique adopted by Eq. (3). Moreover, since the asymptotic diversity order can be determined form the negative slope of the BER curve versus SNR at high SNR on a log-log scale, we can easily find that the achievable diversity order in Rayleigh fading channels becomes 2, as analytically verified in Eq. (4).

4 Conclusion

In this paper, we have derived efficient and simple closed-form BER approximation for AF cooperative relay system in Rayleigh fading channels. By exploiting the formula, we have apparently demonstrated the achievability of cooperative diversity gain and obtained an insight into the asymptotic error rate performance behavior.

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