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Abstract. An optimization method for the optimal sizing of a solar water heating system is presented. The aim of the proposed method is to determine, among a list of commercially available devices in the actual market, the optimal sizing of components ensuring that life cycle cost is minimized subject to the constraint. The genetic algorithm is utilized for optimization design. As an example, we have designed the optimal sizing for a solar water heating system in an office building in Incheon, South Korea. The simulation results show that the developed method can obtain the optimal sizing of the components for a SWHS in a reasonable time.

Keywords: Solar water heating system; Genetic algorithm; Optimal sizing; Life cycle cost.

1 Introduction

Solar thermal is one of the most cost-effective renewable energy technologies and has enormous market potential globally [1]. Solar water heating system (SWHS) is one of the most popular solar thermal systems and accounts for 80% of the solar thermal market worldwide [2].

A critical issue in design for a SWHS is the optimal sizing of the components such as collectors, auxiliary heater and storage tank. Therefore, in recent years, researchers have been developed the some optimization methods i.e., linear and nonlinear optimization method [3-4], design space method [5] and genetic algorithm [6-7] that are conceptually different from the correlation and simulation based methods i.e., the f-chart method and TRNSYS. However, these optimization methods optimized the SWHS comprised of the particular component selected beforehand by researchers. Thus, it’s difficult for them to conduct the optimization design reflecting the economic, technical and environmental characteristics of each device in the actual market.

Therefore, this paper presents a design method to determine the optimal sizing for a SWHS based on an economic criterion using the genetic algorithm while reflecting characteristics of the components and satisfying constraints, such as the energy balance, the space available to install collectors and the solar fraction.
2 Mathematical model of the SWHS

Fig. 1 shows the schematic diagram of a SWHS comprises of a collector array, a hot water storage tank, a heat exchanger and an auxiliary heater.

![Schematic diagram of a solar water heating system.](image)

The energy balance of a well-mixed storage tank can be described as follows [5]:

\[
\frac{dV_s}{dt} = q_s - q_l - q_{ls}.
\]  

where \( T_s \) is the tank temperature; \( \rho, C_{P,w} \) are the density (kg/m\(^3\)) and the specific heat (J/kg°C) of water; \( V_s \) is the volume of the storage tank (m\(^3\)); \( q_s, q_l \) and \( q_{ls} \) are the solar energy supplied to the tank, the heat loss of the tank and the solar energy extracted from the tank, respectively.

The \( q_s \) is the energy transferred from useful heat gain of the collector array according to the differential temperature control, can be calculated as Eq. (2). The \( q_l \) from the tank to the environment surrounding the tank are estimated using Eq. (3).

\[
q_s = m_h C_{P,w} (T_{co} - T_s).
\]

\[
q_l = U_s A_s (T_s - T_e).
\]

where \( T_{co} \) and \( m_h \) are the outlet temperature and the flow rate (kg/s) of cold stream for heat exchanger, \( U_s \) and \( A_s \) are the heat loss coefficient (W/m\(^2\)°C), the surface area (m\(^2\)) of the tank, and \( T_e \) is the temperature surrounding the tank, respectively.

The solar energy supplied from the tank to the load \( (q_{ls}) \) can be estimated using Eq. (4) according to the storage tank temperature. If the tank temperature \( (T_s) \) is greater than the desired hot water temperature \( (T_L) \), the flow rate drawn from the tank \( (m_s) \) is determined by considering the mass and energy balance at the mixing junction \( (m_s = m_L [(T_L - T_r)/(T_s - T_r)]) \). For the opposite case \( (T_s \leq T_L) \), \( m_s \) is discharged at a rate equal to that \( (m_L) \) of the load.

\[
q_{ls} = \begin{cases} 
  m_s C_{P,w} (T_s - T_r), & T_s > T_L \\
  m_L C_{P,w} (T_L - T_r), & T_s \leq T_L
\end{cases}
\]
3 Optimization methodology for SWHS

3.1 Decision variable

Design methodology in this paper is developed to determine the optimal sizing for a SWHS composed of solar collector, auxiliary heater and storage tank. Here, the sizing means the combination of the selected components and is computed using its unit capacity and quantity. Therefore, a SWHS is expressed as a decision vector composed of 5 integer variables that represent the type and number of each component.

3.2 Objective functions

An economic objective function is defined as minimization of life cycle cost (LCC) comprises of the initial cost \( C_I \), the maintenance cost \( C_M \), the replacement cost \( C_R \), the energy cost \( C_E \) and the subsidy cost \( C_S \). It can be formulated as follows:

\[
C_{LCC} = C_I + C_M + C_R + C_E - C_S .
\]  

\[
C_I = (C_{c, j} N_c + C_{aux, j} N_{aux} + C_{s, j})(1 + R_I) .
\]  

\[
C_M = C_R M \left[ \frac{(1+i)^{n_p}-1}{i(1+i)^{n_p}} \right] .
\]  

\[
C_R, c = \sum_{n_{r, c}}^{n_{r, c}} \left( C_{c, j} \left[ \frac{1}{(1+i)(n_{t, c})} \right] \right) .
\]  

\[
C_E = \sum_{t=1}^{t_{max}} C_{ele}(t) UPA_{ele}^* + \sum_{t=1}^{t_{max}} C_{LNG}(t) UPA_{LNG}^* .
\]  

\[
C_S = \begin{cases} 
C_R S, & A_R > A_{c, j} N_c \\
\text{floor}(A_{c, j} C_{c, j}) + C_{aux, j} N_{aux} + C_{s, j}(1 + R_I) R_S, & A_R \leq A_{c, j} N_c 
\end{cases}
\]  

where \( C_{c, j}, C_{aux, j} \) and \( C_{s, j} \) are the purchasing price of the \( j \)th collector, \( j \)th auxiliary heater and \( j \)th storage tank; \( N_{aux} \) is the number of the \( j \)th auxiliary heaters; \( R_I \) is the supplementary cost ratio against the direct purchasing cost; \( R_M \) is the annual maintenance cost ratio of each component; \( n_p \) and \( i \) are the planning period and the real discount rate; \( C_R, c \) and \( C_I, c \) are the replacement and initial cost for each component; \( n_{t, c} \) and \( n_{r, c} \) are the life time of each component, the number of times for replacement; \( C_{ele} \) and \( C_{LNG} \) are the hourly electricity and LNG cost; \( UPA_{ele}^* \) and \( UPA_{LNG}^* \) are the uniform present value factor adjusted to reflect the fuel price escalation rate; \( R_S \) is the subsidy cost’s supporting range; \( A_R \) is the maximum collector array’s area available to receive the subsidy cost, respectively.
3.3 Constraint conditions

In this study, the constraints are categorized into three parts, namely an energy balance, the penetration of solar energy and the available space to install the collector array, which are detailed, respectively as follows.

\[ q_{L,\text{peak}} \leq q_{\text{aux},j}N_{\text{aux}} \quad \text{(11)} \]

\[ F_{S,\text{min}} \leq F_{S} \leq F_{S,\text{max}} \quad \text{(12)} \]

\[ N_{c}W_{c,j}H_{c,j} \left[ \cos \beta + \frac{\sin \beta}{\tan \alpha_{s,w}} \right] \leq A_{c,\text{max}} \quad \text{(13)} \]

where \( q_{L,\text{peak}} \) is the peak load; \( q_{\text{aux},j} \) is the capacity of a certain type of each auxiliary heater; \( F_{S} \), \( F_{S,\text{min}} \) and \( F_{S,\text{max}} \) are a certain, the minimum and maximum solar fraction of the SWHS; \( W_{c,j} \) and \( H_{c,j} \) are the width and height of a certain type of each collector; \( \beta \) is the slope of the collector array; \( \alpha_{s,w} \) is the meridian altitude in winter season, \( A_{c,\text{max}} \) is the available space to install the collectors, respectively.

4 Simulation results and discussion

4.1 Simulation parameters

The proposed method was applied for optimal sizing of a SWHS of an office building located in Incheon, South Korea. The annual hot water load was examined using a data profile of hot water use in office buildings [8], as illustrated in Fig. 2a. The meteorological conditions during the year are illustrated in Fig. 2b.

Fig. 2. Hourly (a) hot water loads (b) meteorological conditions over one year in a case study.

The SWHS of the case study comprise five types of collector, five types of auxiliary heater and five types of storage tank in the commercial marketplace. These
devices can be extended by the designer. Parameters and assumptions required for the optimization process are summarized in Table 1.

Table 1. Parameters and assumptions required for the optimal sizing of SWHS.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope of collector array [°]</td>
<td>30</td>
<td>Project lifetime [years]</td>
<td>40</td>
</tr>
<tr>
<td>Azimuth of collector array [°]</td>
<td>0</td>
<td>Real discount rate [%]</td>
<td>2.91</td>
</tr>
<tr>
<td>Meridian altitude in winter [°]</td>
<td>29</td>
<td>Nominal interest rate [%]</td>
<td>6.00</td>
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<tr>
<td>Ground reflectance [-]</td>
<td>0.2</td>
<td>Inflation rate [%]</td>
<td>3.00</td>
</tr>
<tr>
<td>Minimum solar fraction [%]</td>
<td>10</td>
<td>Electricity cost escalation rate [%]</td>
<td>4.00</td>
</tr>
<tr>
<td>Maximum solar fraction [%]</td>
<td>60</td>
<td>Gas cost escalation rate [%]</td>
<td>4.00</td>
</tr>
<tr>
<td>Area available to install SWHS [m²]</td>
<td>500</td>
<td>Supplementary cost ratio [%]</td>
<td>30</td>
</tr>
<tr>
<td>Maximum collector’s area [m²]</td>
<td>500</td>
<td>Maintenance cost ratio of SWHS [%]</td>
<td>1.5</td>
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<tr>
<td>Life time of collector [year]</td>
<td>20</td>
<td>Maximum supporting range [%]</td>
<td>50</td>
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<tr>
<td>Life time of auxiliary heater [year]</td>
<td>15</td>
<td>Emission by electricity [kgCO₂eq/MWh]</td>
<td>468.92</td>
</tr>
<tr>
<td>Life time of tank [year]</td>
<td>10</td>
<td>Emission by LNG [kgCO₂eq/MWh]</td>
<td>202.45</td>
</tr>
</tbody>
</table>

4.2 Results of the optimization design for SWHS

Using a computer with the Windows 7 operating system, Intel(R) Core(TM) i5-2310 @2.90GHz CPU, and 4 GB memory, developed method has consumed about 30 seconds to find the optimal solution. The genetic algorithm [9] parameters selected for this case study are the following: number of generation and population are 100 and 40; crossover and mutation probability are 0.9 and 0.7, respectively.

Fig. 3 shows the variation of the LCC of the best and worst solutions. It can be seen that there are a wide range of values for objective function at the beginning of the optimization, which mainly identified and assessed the various possible SWHSs. In the subsequent generations, the probabilities of identifying new solutions that are able to improve the LCC decrease and most solutions of the parent population reform the offspring population. Table 2 shows characteristics of the optimal SWHS for the case study. The optimal SWHS with LCC of about 383 million KRW is comprised of 60 collectors (169.8m²), 2 heaters (58.16kW) and a storage tank of 2.65 m³.

Fig. 3. Variation of the objective functions (LCC) for each generation.
Table 2. Characteristics of the optimal SWHS for the case study.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Unit area of a collector [m²]</td>
<td>2.83</td>
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<tr>
<td>Number of collectors [ea]</td>
<td>60</td>
</tr>
<tr>
<td>Area of collector array [m²]</td>
<td>169.80</td>
</tr>
<tr>
<td>Unit capacity of auxiliary heater [kW]</td>
<td>29.08</td>
</tr>
<tr>
<td>Number of auxiliary heaters [ea]</td>
<td>2</td>
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<tr>
<td>Capacity of auxiliary heaters [kW]</td>
<td>58.16</td>
</tr>
<tr>
<td>Volume of a storage tank [m³]</td>
<td>2.65</td>
</tr>
<tr>
<td>Peak hot water load [kW]</td>
<td>51.97</td>
</tr>
<tr>
<td>Life cycle cost [million KRW]</td>
<td>383.03</td>
</tr>
<tr>
<td>Annual solar fraction [%]</td>
<td>57.97</td>
</tr>
</tbody>
</table>

5 Conclusions

This paper has presented a design method to determine the optimal sizing for a SWHS based on LCC using the genetic algorithm. As an application example, we carried out a design for determining the optimal sizing of a SWHS for an office building in Incheon, South Korea. With the case study, the optimal solution was obtained from a number of possible solutions within a reasonable computation time. Future work includes applying the technical and environmental objectives as multiple conflicting objectives and sensitivity study of parameters such as the energy charge, the pattern and magnitude of hot water load.

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References