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Algorithm for Multivariable System Reduction using Generalized Block Pulse Function

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Abstract. In most case, the actual physical system is represented by a high order model. According to the system order increase, system analysis and controller design becomes more complex and difficult. Therefore an attempt to obtain a low order model with the dynamic characteristics similar to a high order system is needed. An algorithm for multivariable system approximation and reduction based on the generalized block pulse function is presented in this paper. The algorithm adopted in this paper is shown that computational results are more accurate and convenient rather than conventional block pulse function.

Keywords: multivariable system**,** high order model, reduction, generalized block pulse functions, operational matrix

1 Introduction

A control system may have been described by a high order differential equation or transfer function and the definition of transfer function is extended to a multivariable system that has multiple inputs and outputs as shown in figure 1. Unlike a single input and single output system, a multivariable system can be represented as matrix polynomial form and it is more complex to handle with. In a multivariable system, a differential equation may be used to describe the relationship between a pair of input and output variables, when all other inputs are set to zero. The conventional block pulse function is one of the basic orthogonal function set that was introduced by Harmuth in 1969[1]. Meanwhile, the generalized block pulse operational matrix was derived to replace the repeated use of the conventional block pulse operational matrix. In this paper, a multivariable high order system is transformed into its corresponding algebraic expression and is replaced with a lower order model using generalized block pulse function. The results of the proposed algorithm are more accurate and convenient rather than conventional block pulse function.

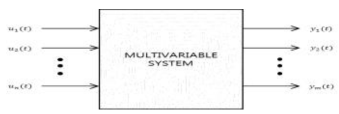

Fig. 1. Block diagram of multivariable system

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2 Conventional and Generalized Block Pulse Function

Block pulse functions are a set of orthogonal functions with piecewise conatant value and are usually applied as a useful tool in the system analysis, design and other problems of control engineering. This set of functions was first introduced in 1969.[2] The conventional block pulse function $cB_i(\lambda)$ is usually defined in the unit interval $\lambda \in [0, 1)$ as

$$cB_i(\lambda) = \begin{cases} 1, & \frac{(i-1)}{m} \le \lambda \le \frac{i}{m} \\ 0, & otherwise \end{cases} \quad (1)$$

It has only one rectangular pulse of unit height. But if we use the substitution $t=T\lambda$, we can obtain a similar definition of the block pulse function with an interval of arbitrary length with each other in the interval $t \in [0, T)$. The block pulse functions are orthogonal with each other in the interval $t \in [0, T)$:

$$\int_0^T cB_i(t) cB_j(t) dt = \begin{cases} h, & for\ i=j \\ 0, & for\ i \ne j \end{cases} \quad (2)$$

where $i, j = 1, 2 \ldots .m$. It is well known that a function $f(t)$ which is integrable in $(0, 1)$ can be approximated and expanded as

$$f(t) \cong \sum_{i=1}^{m} f_i cB_i(t) \quad (3)$$

We can expand the integral of block pulse function into the set of basic function with the operational matrix. In case of generalized block pulse function, we can obtain as

$$\int_0^1 gB(t) dt \cong P_{gb} gB(t) \quad (4)$$

where $gB(t)$ is generalized block function and P_{gb} is its operational matrix. The generalized block function operational matrix P_{gb} was derived by Wang in 1982. It was shown that the conventional block pulse operational matrix is a special case of the generalized block pulse operational matrix with $k=0$.[3] Furthermore, in case of repeated integration of a function, the generalized block pulse operational matrix shows more accurate results than the conventional block pulse operational matrix case. And the coefficients of expanded function $f(f)$ are determined such that following integral square error ε is minimized as[4]

$$\varepsilon = \int_0^t [f(t) - \sum_{i=1}^{m} f_i gB(t)]^2 dt \quad (5)$$

$$P_{gb} = (\frac{T}{m})^{k+1} \frac{1}{(k+2)!} \begin{bmatrix} f_1 & f_2 & f_3 & \cdots & f_m \\ 0 & f_1 & f_2 & \cdots & f_{m-1} \\ 0 & 0 & f_1 & \cdots & f_{m-2} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & f_1 \end{bmatrix} \quad (6)$$

3 Reduction Algorithm using Generalized Block Pulse Function

Let's consider a linear system which is described by a transfer function and express the output *y(t)* using generalized block pulse function.

$$G(s) = \frac{Y(s)}{U(s)} = \frac{a_1 s^{m-1} + a_2 s^{m-2} + \cdots + a_{m-1}s + a_m}{s^n + b_1 s^{n-1} + \cdots + b_{n-1}s + b_n} \tag{7}$$

With all the initial conditions are set to zero, and *Y(s)* and *U(s)* are Laplace transforms of *y(t)* and *u(t)*, respectively. The output variable $y_i(t)$ and input variable $u_i(t)$ can be expressed using the generalized block pulse function and its operational matrix.[5]

$$y_i(\text{t}) = \sum_{i=1}^{n} C_i {P_{gb}}^i gB(t) = C^T P_{gb} gB(t) \tag{8}$$

$$u_i(\text{t}) = \sum_{i=1}^{n} D_i {P_{gb}}^i gB(t) = D^T P_{gb} gB(t) \tag{9}$$

The *C* is generalized block pulse function coefficient vector of the output and the D is input coefficient vector. The *T* denotes transpose. And we use the substitution $t = ⁊\lambda$, then we can obtain a similar definition of the generalized block pulse function with an interval of arbitrary length and apply generalized block pulse function to equation (8) and (9) for transform.[6]

$$\sum_{i=1}^{n} b_i D_i^T ⁊^i P_{gb}^i gB(t) = C^T gB(⁊) + \sum_{i=1}^{n} a_i C_i^T ⁊^i P_{gb}^i gB(t) \tag{10}$$

Now we can express transfer function of reduced model with *p* inputs and *q* outputs. In this case, the definition of a transfer function is easily extended to a multivariable system with multiple inputs and outputs. So we can express lower order multivariable transfer function $\breve{G}_M(s)$ in matrix form.

$$\breve{G}_M(s) = \frac{\breve{Y}(s)}{\breve{U}(s)} = \frac{\breve{a}_1 s^{r-1} + \breve{a}_2 s^{r-2} + \cdots + \breve{a}_{f-1}s + \breve{a}_f}{\breve{b}_1 s^{l-1} + \breve{b}_2 s^{l-1} + \cdots + \breve{b}_{h-1}s + \breve{b}_h} \tag{11}$$

$$\breve{G}_M(s) = \begin{bmatrix} L_{11}(s) & L_{12}(s) & \cdots & L_{1p}(s) \\ L_{21}(s) & L_{22}(s) & \cdots & L_{2p}(s) \\ \vdots & \vdots & \cdots & \vdots \\ L_{q1}(s) & L_{q2}(s) & \cdots & L_{qp}(s) \end{bmatrix} \tag{12}$$

where *m*>*r* and *n*>*l*. In general, if a reduced multivariable system has *p* inputs and *q* outputs, the transfer function between *j*th input and *i*th output is defined as

$$L_{ij}(\text{s}) = \frac{\breve{Y}_i(s)}{\breve{U}_j(s)} \tag{13}$$

where $\breve{U}_k(s) = 0, k = 1, 2, \ldots, p, k \neq j$.
Now we can apply equation (10) to determine generalized block pulse function coefficient vector $\breve{C}$ and $\breve{D}$ then the coefficient vector can be obtained as[7]

$$\sum_{i=1}^{r} \breve{b}_i \breve{D}_i^T \mathrm{T}^i P_{gb}^i = \breve{C}^T + \sum_{i=1}^{r} \breve{a}_i \breve{C}_i^T \mathrm{T}^i P_{gb}^i \quad (14)$$

Therefore we can define the $L_{ij}(\mathrm{s})$ of reduced transfer function of multivariable model using generalized block pulse function and operational matrix. For example, let's apply the suggested algorithm to a high order multivariable system with three inputs and outputs. So *G(s)* of the example has *3x3* matrix but just applied G_{11} and G_{33} for a simple example presented in this case.

$$G(s) = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \quad (15)$$

$$G_{11} = \frac{1}{0.122s^3+0.831s^2+1.43s+1}, \quad (16)$$

$$G_{33} = \frac{1}{0.04s^3+0.58s^2+1.07s+1} \quad (17)$$

As a result, we can obtain the reduced multivariable transfer function $\breve{G}_M(s)$ of equation (18) by applying proposed algorithm based on generalized block pulse function and operational matrix. In equation (18), the original third-order system is reduced to first-order system as shown in L_{11} and L_{33}.

$$\breve{G}_M(s) = \begin{bmatrix} L_{11} = \frac{1}{0.854s+1} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} = \frac{1}{1.98s+2.01} \end{bmatrix} \quad (18)$$

This method is useful to determine lower order multivariable system from original high order model.

4 Conclusions

In control theory if the order of system can be reduced, system analysis and design is easier than the original high order model cases. But the reduced first-order model gives a more inferior approximation to the original third-order system than the second-order system. In this paper, generalized block pulse functions and operational matrix are used to transform the multivariable transfer function. The proposed algorithm is simple and useful.

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