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The Interval Sequence Decision Method for Multi-attribute Science Award

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Abstract. Science achievement is combinations with lots of attributes, such as creativity, sociality and scientificalness, etc. And process of judgement is a fuzzy synthesis in science award decision, so it is difficult to model them in conventional figures. It makes a conclusion that science achievement and decision results are showed in interval fuzzy numbers reasonably and scientifically after research attributes of interval number. Therefore, in the paper sequence between interval numbers is developed and further study points to properties of interval sequence. On the basis of properties, we prove that there must be the interval sequence between interval fuzzy numbers and the interval sequence is exclusive theoretically. Consequently, science achievement could be ranked by interval sequence function. At last, we present interval fuzzy sets to scores of experts and to model of science award, and the decision is reasonable.

Keywords: interval fuzzy number, order, decision, interval sequence

1 Introduction

How to decide is very popular in the social life, engineering project and economic system. Decision is fuzzy and uncertainty for that event, project, or system is blur and undefined, so model decision system in interval fuzzy numbers is more appropriate than in conventional numbers. Such as there are ten experts scored science achievement: 91, 92, 88, 89, 90, 93, 89, 92, 90, 91, and the mean data, 90.5 will be deemed to the final score generally. Another science achievement is scored: 94, 90, 91, 90, 90, 90, 90, 90, 90, 90, and 90.5 is the mean number too. The conventional mean score is too simple to order two projects obviously and misses lots of helpful information. In addition, for vagueness of human thinking, it is difficult that to describe the judgment with clear and exclusive number. So it is appropriate to model scientific judgment with interval fuzzy numbers, such as interval fuzzy numbers [88, 92] and [90, 94] represent two science projects above. Fuzzy numbers [88, 92] and [90, 94] are completely different.

That is to say, the key issue of decision making is how to model events in interval fuzzy numbers and how to sort interval fuzzy numbers. There are two kinds of sorting method of interval fuzzy numbers, similarity measure and ranking. Sorting interval

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fuzzy numbers by their similarity is restricted in using, so there is a few studies on similarity measure (see [1] and [2]). However there are a lot of findings on how to rank interval fuzzy numbers, such as Moore defined the relation of size between two interval fuzzy numbers in 1979 in [3], and weak preference ideas was proposed by Ishibuchi and Tanaka to determine two interval fuzzy numbers size in [4]. These are studies on interval fuzzy number sequence early, but there was no methodology in methods. Later, Japanese scholars Nakahara defined possibility degree formula to compare two interval fuzzy numbers by [5]; however he couldn't demonstrate the formula by rigorous theory unfortunately. In 1997, a partial order relation of interval numbers was put forward on minimum transfer of fuzzy left relationship by Kundu in [6], which is so complex to calculate. In China, there are amounts of interval fuzzy number research, but these studies are not systematic and theoretical. Therefore, in this paper, we briefly discuss rationality of interval sequence existence, then form the notion of interval sequence and further research its properties to present method for solving the interval sequence. At last, we prove the rationality of the sorting method in theory and the method developed is tested by judgment of science award, and results show that it is reasonable and practicable.

2 Fundamental Concepts

Definition2.1：As a generalization of ordinary fuzzy number, the notion of interval fuzzy number is defined newly in the paper. In the sequel, let

$$\left[R^+\right]=\left\{\left[a^-,a^+\right]:a^-\le a^+,a^-,a^+\in R^+\right\}$$

be the family of all closed positive interval fuzzy number in which R^+ represents the positive real number set and all interval fuzzy numbers form the set $I(R^+)$.

When $a^-=a^+$, interval fuzzy number a would be the common real number, so we can conclude that the real set R^+ is a special case of interval number set in $I(R^+)$.

Definition2.2：As we know, set formed from interval fuzzy numbers is no order relation. In order to solve the problem of multiple attribute decision making using a variety of methods, we define sequence relationship between interval fuzzy numbers, which is called interval sequence briefly. There are 3 sorts of interval sequence, superior, inferior and equal, represented by symbol >, < and =, or merged into $\ge$ and $\le$.

For any $a,b,c\in I(R^+)$, interval sequence accords with properties of dyadic relation as follows:

Property 1: Equal sequence is reflexive, that is to say $a=a$;

Property 2: Interval sequence is transitive. That is, if $a<b$ and $b<c$, then we can conclude that $a<c$;

Property 3: There is interval sequence between interval fuzzy number a and b certainly, and there is only a sort of relationship in >, <, = , $\ge$ or $\le$;

Property 4: Interval sequence is linear, i.e. $\forall\lambda\in R^+$, if $a<b$ then $a+c<b+c$ and $\lambda a<\lambda b$.

3 Interval Sequence Function

Definition3.1: Let $a,b \in I(R^+)$, M_a is defined as the core of interval fuzzy number a, $M_a = \frac{1}{2}(a^+ + a^-)$; and W_a is called the radius of interval fuzzy number a, $W_a = \frac{1}{2}(a^+ - a^-)$; next interval sequence function is showed as follows:

$f_a(\lambda) = (M_a - W_a) + 2\lambda W_a$, in which $\lambda \in [0,1]$.

Property 3.1: For any $\lambda \in [0,1]$, let $a,b \in I(R^+)$, (1) $f_a(0) = a^-$, $f_a(0.5) = M_a$, $f_a(1) = a^+$; (2) $f_a(\lambda) = f_0(\lambda) + 2w_a\lambda$; (3) If $0 \le \lambda_1 \le \lambda_2 \le 1$, then $f_a(\lambda_1) \le f_a(\lambda_2)$; (4) If $a = b$, then $f_a(\lambda) = f_b(\lambda)$; (5) $\frac{d}{d\lambda} f_a(\lambda) = 2W_a$, derivative of interval sequence function is the diameter of interval fuzzy number; (6) $\int_0^1 f_a(\lambda)d_\lambda = M_a$, integration of the interval sequence function is the core of interval fuzzy number.

Theorem 3.1: Let $a,b \in I(R^+)$, $\forall \lambda \in R^+$, If $f_a(\lambda) < f_b(\lambda)$, then $a < b$; If $f_a(\lambda) = f_b(\lambda)$, then $a = b$.

Proof. There are 3 kinds of binary relation between interval fuzzy number a and b from number sets, empty set, intersection, and including after intersected, showed in ①, ② and ③.

① $a \cap b = 0$, that is $a^+ < b^-$;

② $a \cap b \ne 0$, that is $a^- < b^- < a^+ < b^+$;

In ① and ②, compare functions f of a and b. $f_a(\lambda) - f_b(\lambda) = (a^- - b^-)(1+\lambda) + \lambda(a^+ - b^+)$, for any $\lambda \in [0,1]$, $f_a(\lambda) < f_b(\lambda)$ is true. Therefore theorem 3.1 in case ① and ② is proved right in accordance with of weak partial relation defined by Ishibuchi and Tanaka in [4] on 1990.

The last binary relation is including as follows:

③ $a \subseteq b$, that is $b^- \le a^- \le a^+ \le b^+$, $M_a \ge M_b$ and $W_a \ge W_b$;

Calculating $f_a(\lambda) - f_b(\lambda) = (a^- - b^-) + \lambda[(a^+ - b^+) - (a^- - b^-)]$, for $W_a \le W_b$, then $(a^+ - b^+) - (a^- - b^-) \le 0$, and for $(a^- - b^-) \le 0$, we can easily obtain that $f_a(\lambda) \le f_b(\lambda)$.

So theorem 3.1 under ③ is justified which is defined as $a \le b$ in [1].

Theorem 3.2: Let $a,b \in I(R^+)$ be any two closed interval fuzzy numbers, we define $\lambda_k \in [0,1]$ as decision level of confidence. If $\lambda_k > 0.5$, more than half of the people will agree to order results and it is acceptable for decision makers. For example, when more than 5 men agree in 10 voters, then results of ranking is what trust.

For any $\lambda \in [0,1]$ and $\lambda_k \in [0,1]$, (1) When $0 \le \lambda < \lambda_k$ and $f_a(\lambda) < f_b(\lambda)$, then $p\{a < b\} = p\{0 \le \lambda < \lambda_k\} = \lambda_k$; (2)When $\lambda = \lambda_k$ and $f_a(\lambda) = f_b(\lambda)$, then $p\{a = b\} = p\{\lambda = \lambda_k\} = \frac{1}{n}$, in which n is the length of λ.

4 Interval Fuzzy Sequence Judgment Model

Set $X = \{x_1, x_2, ..., x_n\}$ be a set of properties for judgment, where x_i is index of judgment, e.g. x_i stands for theoretical value or application value and so on. Let w be weight of judgment, and then we have effective weight set $w = \{w, w, ..., w\}^T$.

We consider a interval fuzzy numbers matrix A, where $a_{ln} = [a_{ln}^-, a_{ln}^+]$ is the element in A, and where expert is named by n and attribute is called l .

$$A = \begin{bmatrix} A_1 \\ A_2 \\ \dots \\ A_l \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ & \dots & \dots & \\ a_{l1} & a_{l2} & \dots & a_{ln} \end{bmatrix} = \begin{bmatrix} [a_{11}^-, a_{11}^+] & [a_{12}^-, a_{12}^+] & \dots & [a_{1n}^-, a_{1n}^+] \\ [a_{21}^-, a_{21}^+] & [a_{22}^-, a_{22}^+] & \dots & [a_{2n}^-, a_{2n}^+] \\ & \dots & \dots & \\ [a_{l1}^-, a_{l1}^+] & [a_{l2}^-, a_{l2}^+] & \dots & [a_{ln}^-, a_{ln}^+] \end{bmatrix}$$

The peck order between A_i and A_j is decided by interval sequence function $f_{A_i}(\lambda)$ and $f_{A_j}(\lambda)$, in which $f_{A_i}(\lambda) = \sum_{j=1}^{n} w_j f_{a_{ij}}(\lambda)$, $i = 1, 2, ..., l$. The function $f_{a_{ij}}(\lambda)$ is well known defined in previous definition 3.1 and w_j is the weight of A_i .

5 Science Award Judgment Case by Interval Sequence

To take one example, the current standard of judgment is showed by $X = \{x_1, x_2, x_3, x_4, x_5\}$ in Daqing in China, where

x_1 is difficult and hot in the current international economic and social development; x_2 is full and accurate information about research project; x_3 is exactly and profound analysis; x_4 is operational suggestion and measures, etc; x_5 is high theoretical and practical value.

Let $W = \{w_1, w_2, w_3, w_4, w_5\}^T == \{0.1, 0.2, 0.2, 0.2, 0.3\}$ be attribute weight given by experts.

There are 10 experts to order 6 projects. Experts score to attributes in interval fuzzy numbers for each project as table 1.

Table 1. Interval Scores of Scientific Achievements

	x_1	x_2	x_3	x_4	x_5
A_1	[90, 95]	[92, 94]	[89, 92]	[90, 94]	[80, 92]
A_2	[82, 83]	[80, 84]	[85, 86]	[83, 85]	[82, 89]
A_3	[92, 93]	[90, 95]	[92, 93]	[91, 95]	[89, 92]
A_4	[70, 72]	[72, 73]	[71, 72]	[73, 75]	[73, 74]

A_5	[60, 68]	[62, 69]	[60, 62]	[61, 63]	[61, 69]
A_6	[75, 76]	[73, 77]	[72, 75]	[71, 73]	[70, 71]

Further calculations, interval sequence function with weights are listed in Table 2.

Table 2. Value of Interval Sequence Function

$f_{A_1}(\lambda)$	$f_{A_2}(\lambda)$	$f_{A_3}(\lambda)$	$f_{A_4}(\lambda)$	$f_{A_5}(\lambda)$	$f_{A_6}(\lambda)$
$87.2+5.9\lambda$	$82.4+3.6\lambda$	$90.5+3\lambda$	$72.1+1.3\lambda$	$60.9+5.4\lambda$	$71.7+2.2\lambda$

At first, suppose score of project A_1 is highest and compare interval sequence function of A_1 with others. Clearly $\forall\lambda\in[0,1]$, $f_{A_1}(\lambda)-f_{A_2}(\lambda)=4.8+2.3\lambda>0$ is true at all time, therefore we can judge that $A_1>A_2$ by theorem 3.2; Next $f_{A_1}(\lambda)-f_{A_3}(\lambda)=-3.3+2.9\lambda$, for any $\lambda\in[0,1]$, there is $f_{A_1}(\lambda)-f_{A_3}(\lambda)<0$, that is to say $A_1<A_3$. So we give up original maximum A_1 to reassign A_3 . Comparing interval sequence function of A_3 with others as usually, we get A_3 is the first finally.

Calculated by $f_{A_4}(\lambda)-f_{A_6}(\lambda)=0.4-0.9\lambda=0$, $\lambda_k=0.44$. Thus,

$P\{A_4<A_6\}=1-P\{A_4\geq A_6\}=1-P\{0\leq\lambda\leq\lambda_k\}=0.56$.

That is to say A_6 is in front of A_4 .

Therefore the total order is A_3 , A_1 , A_2 , A_6 , A_4 .

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