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Synchronization control of Fractional-order Hyperchaotic Systems

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Abstract. An active adaptive sliding mode controller is investigated for synchronization of fractional-order hyperchaotic systems with uncertainties and external disturbances in the study. A fractional sliding surface is designed and an active adaptive sliding mode controller is designed using Lyapunov stability theorems. In this method, the bound of the uncertainties and external disturbances are estimated through the adaptive updating law. The synchronization scheme is global and theoretically rigorous. Simulation studies have shown the proposed controllers can get good control effects.

Keywords: fractional-order hyperchaotic system, synchronization, anti-synchronization, active adaptive sliding model, uncertainties and external disturbances

1 Introduction

Recently, synchronization of fractional-order chaotic systems starts to receive increasing attention due to its potential applications in secure communication and control processing [1]. Hyperchaotic system has more complex dynamical behaviors than chaotic system and is more suitable in secure communications [2]. So study on hyperchaotic synchronization is very important and is a more challenging research [3-4].

So far only few synchronization methods such as feedback controller and nonlinear controller have been proposed [5-6]. Most of the above synchronization methods seldom deal with uncertainties and external disturbances. The existence of uncertainties and external disturbances is often the cause of poor performance, undesirable system transient response, and instability. Therefore, in this paper, we present a new fractional order controller to synchronize the hyperchaotic systems with uncertainties and external disturbances. The adaptive updating law is designed to estate the bound of the uncertainties and external disturbances under the combination of active sliding mode control and adaptive control. Synchronization and anti-synchronization are converted via changed value of the parameter in the control function. The stability of error dynamics are demonstrated based on the Lyapunov stability theory. Numerical simulation of hyperchaotic system illustrates the effectiveness and superiority of the proposed control method.

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2 Active adaptive sliding mode controller design and analysis

Consider the following drive-response systems described by (1) and (2)

$$D_t^{\alpha} x = (A_1+\Delta A_1)x + f_1(x) + \delta_1(t) \tag{1}$$

and

$$D_t^{\alpha} y = (A_2+\Delta A_2)y + f_2(y) + \delta_2(t) + u(t) \tag{2}$$

where $0 < \alpha < 1$.

Suppose

$$e = y + \beta x \tag{3}$$

The error system can be rewritten as

$$D_t^{\alpha} e = (A_2+\Delta A_2)y + f_2(y) + \delta_2(t) + u(t) + \beta((A_1+\Delta A_1)x + f_1(x) + \delta_1(t)) + u(t) = (A_2 + \Delta A_2 + \Delta A_1)e + \delta_2(t) + \beta\delta_1(t) + f_2(y) + \beta f_1(x) + \beta(A_1 - (A_2+\Delta A_2))x - \Delta A_1 y + u(t) \tag{4}$$

For simplicity, (4) is expressed as:

$$D_t^{\alpha} e = (A_2+\Delta A_2 + \Delta A_1)e + \delta_2(t) + \beta\delta_1(t) + F(x,y) + u(t) \tag{5}$$

where $F(x,y) = f_2(y) + \beta f_1(x) + \beta(A_1 - (A_2+\Delta A_2))x - \Delta A_1 y + u(t)$.

Assumption 1. $\|\Delta A_2 + \beta\Delta A_1\| < \psi$ and $\|\delta_2 + \beta\delta_1\| < \eta$,where ψ, η are unknown positive constants.

According to the active control design procedure, the $u(t)$ is considered as

$$u(t) = G(t) - F(x,y) \tag{6}$$

There are many possible methods for the control input *G(t)*. Without loss of generality, we choose the sliding mode control law as follows:

$$G(t) = Kv(t) \tag{7}$$

where $K = [k_1, k_2, \cdots, k_{n-1}, k_n]^{\mathrm{T}}$ is a constant gain vector .

The sliding surface in the error state space is defined as:

$$s = D_t^{\alpha-1} C e \tag{8}$$

where $C = [C_1, C_2, \cdots, C_{n-1}, C_n]^{\mathrm{T}}$ is a constant vector.

When in sliding surface, the controlled system should satisfy the following conditions:

$$s = D_t^{\alpha-1} C e = 0 \text{ and } \dot{s} = Ds = CD_t^{\alpha} e = 0 \tag{9}$$

The reaching law is expressed as:

$$\dot{s} = -\varepsilon\,\mathrm{sgn}(s) - rs \tag{10}$$

Hence, the $v(t)$ can be expressed as follow:

$$v(t) = -(CK)^{-1}[CA_2e + C\Delta A_2y + \beta C\Delta A_1x + C\delta_2(t) + \beta C\delta_1(t) + rs + \varepsilon\cdot\mathrm{sgn}(s)] \tag{11}$$

where the existence of $(CK)^{-1}$ is a necessary condition.

There exist system uncertainties and external disturbances in (11). In this regard, we propose the following control law as follow:

$$v(t) = -(CK)^{-1}[CA_2e + \|C\|\hat{\psi}\,\mathrm{sgn}(s) + \|C\|\hat{\eta}\,\mathrm{sgn}(s) + rs + \varepsilon\cdot\mathrm{sgn}(s)] \tag{12}$$

The suitable adaptive laws are defined as follow:

$$\dot{\hat{\psi}} = -\|Cs\|, \dot{\hat{\eta}} = -\|Cs\| \tag{13}$$

To prove that the error dynamics (4) is asymptotically stable, we choose the Lyapunov function defined by the equation

$$V = \frac{1}{2}s^2 + \frac{1}{2}\tilde{\psi}^2 + \frac{1}{2}\tilde{\eta}^2 \tag{14}$$

Obviously, V is a positive definite function on R^n, where $\tilde{\psi} = \hat{\psi} + \psi, \tilde{\eta} = \hat{\eta} + \eta$.

$$\begin{aligned}
\dot{V} &= s\dot{s} + \tilde{\psi}\dot{\tilde{\psi}} + \tilde{\eta}\dot{\tilde{\eta}} = sC(A_2e - K(CK)^{-1}(CA_2e + \|C\|\hat{\psi}\,\mathrm{sgn}(s) \\
&+ \|C\|\hat{\eta}\,\mathrm{sgn}(s) + \varepsilon\,\mathrm{sgn}(s) + rs) + \Delta A_2y + \beta\Delta A_1x + \delta_2(t) + \beta\delta_1(t)) + \tilde{\psi}\dot{\tilde{\psi}} + \tilde{\eta}\dot{\tilde{\eta}} \\
&= -\varepsilon s\,\mathrm{sgn}(s) - rs^2 - \|C\|\hat{\psi}s\,\mathrm{sgn}(s) - \|C\|\hat{\eta}s\,\mathrm{sgn}(s) + sC(\Delta A_2y + \beta\Delta A_1x) \\
&+ sC(\delta_2(t) + \beta\delta_1(t)) + \tilde{\psi}\dot{\tilde{\psi}} + \tilde{\eta}\dot{\tilde{\eta}} \le -\varepsilon s\,\mathrm{sgn}(s) - rs^2 - \|C\|\hat{\psi}s\,\mathrm{sgn}(s) - \\
&\|C\|\hat{\eta}s\,\mathrm{sgn}(s) + \|Cs\|\|\Delta A_2y + \beta\Delta A_1x\| + \|Cs\|\|\delta_2(t) + \beta\delta_1(t)\| + \tilde{\psi}\dot{\tilde{\psi}} + \tilde{\eta}\dot{\tilde{\eta}} \le \\
&-\varepsilon s\,\mathrm{sgn}(s) - rs^2 - \|C\|\hat{\psi}s\,\mathrm{sgn}(s) - \|C\|\hat{\eta}s\,\mathrm{sgn}(s) + \|Cs\|\psi + \|Cs\|\eta + \tilde{\psi}\dot{\tilde{\psi}} + \tilde{\eta}\dot{\tilde{\eta}} \\
&= -\varepsilon s\,\mathrm{sgn}(s) - rs^2 - \|Cs\,\mathrm{sgn}(s)\|\hat{\psi} - \|Cs\|\hat{\eta} - \|Cs\|\hat{\psi} - \|Cs\|\hat{\eta} \le 0
\end{aligned} \tag{15}$$

Therefore the two systems (1) and (2) are globally asymptotically synchronized.

3 Numerical simulation

The fractional order hyperchaotic Chen drive systems system is written as

$$\begin{cases} D_t^{\alpha} x_1 = 35(x_2 - x_1) + x_4 + 3x_1 + x_2 + x_4 - 0.5\cos(50t) \\ D_t^{\alpha} x_2 = 7x_1 - x_1 x_3 + 12x_2 - 2x_2 + 0.5\sin(50t) \\ D_t^{\alpha} x_3 = x_1 x_2 - 3x_3 + x_1 + \sin(50t) \\ D_t^{\alpha} x_4 = x_2 x_3 + 0.5x_4 + x_3 - \sin(50t) \end{cases} \tag{16}$$

The fractional order hyperchaotic Chen response system is written as

$$\begin{cases} D_t^{\alpha} y_1 = 35(y_2 - y_1) + 2y_4 + 2y_1 + y_2 - 0.5\cos(50t) + u_1(t) \\ D_t^{\alpha} y_2 = 7y_1 - x_2 y_3 + 12y_2 - 2y_2 + 0.3\sin(50t) + u_2(t) \\ D_t^{\alpha} y_3 = y_1 y_2 - 3y_3 + 2y_1 - 0.2\sin(50t) + u_3(t) \\ D_t^{\alpha} y_4 = y_2 y_3 + 0.5y_4 + y_3 - 0.2\sin(50t) + u_4(t) \end{cases} \tag{17}$$

The corresponding numerical results are shown in Fig. 1, where the initial values are set as $\beta = [\beta_1, \beta_2, \cdots \beta_{n-1}, \beta_n]^{\mathrm{T}} = [-1, -1, \cdots -1, -1]^{\mathrm{T}}$, $x(0) = (1, 5.8, -1, 2)$, $y(0) = (2, 2, -1, 2)$, $\psi(0) = 28$, $\eta(0) = -31, C = (1,\ 2,\ 1,\ -1)$ and $K = (1,\ 1,\ 0,\ 1)^{\mathrm{T}}$.

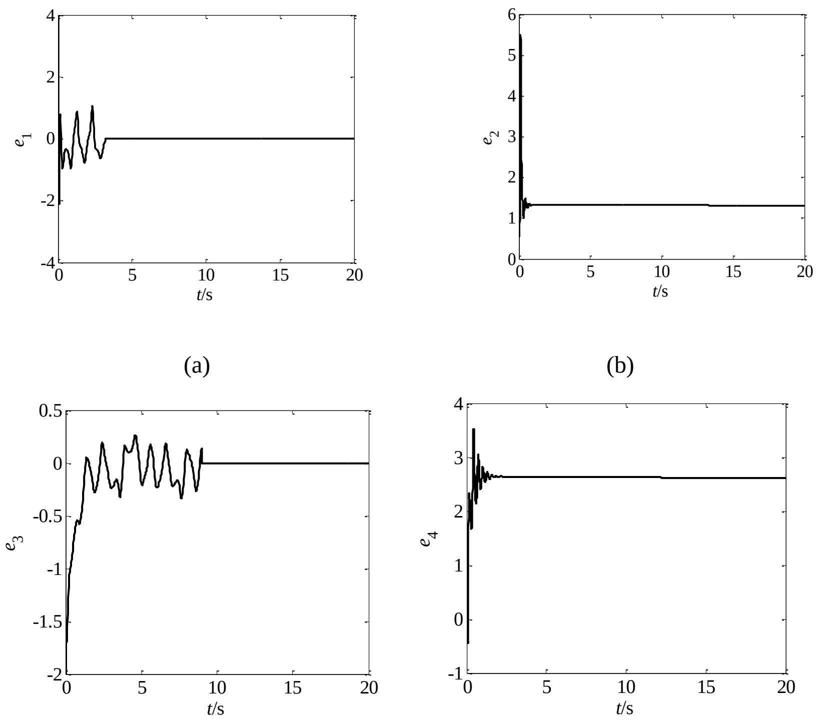

Fig.1. Synchronization error of the Chen systems

5 Conclusions

In this paper, based on Lyapunov stability theorems an active adaptive sliding mode controller has been proposed to synchronize fractional-order hyperchaotic systems with uncertainties and external disturbances. The proposed controller is simple and theoretically rigorous. Simulation results have illustrated the effectiveness of the theorem and the proposed method.

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