Research on the multivariable predictive functional control algorithm based on the state space equation

Zhenyu Lu¹, Weihan Zhao¹, Tingya Yang², Wei Guo³, Jin Wang³

¹ School of Electrical and Information Engineering, Nanjing University of Information Science & Technology, Nanjing 210044, China
² Jiangsu Meteorological Observatory, Nanjing 210008, China
³ School of Information and Control, Nanjing University of Information Science & Technology, Nanjing 210044, China

Abstract. The research and application of predictive functional control is mainly focused on single-variable system. The actual object is multivariable systems. So it is necessary to research a predictive functional control for multivariable systems. This paper mainly studied the basis function, reference trajectory, and the output of the predictive model. Finally, The expressions of control of the multivariable predictive function control algorithm can be deduced in this paper. The state space equations are adopted by the controlled object, and then the multivariable system is simulated by MATLAB. The simulation results show that the control algorithm of MPFC can achieve better control effect and quickly make the system get a stable state, but some overshoot is appeared.

Keywords: MPFC, Basis function, Reference trajectory, Predictive model, The state space equation

1 Introduction

The research and application of predictive functional control is mainly focused on single-variable system. The actual object is multivariable systems. So it is necessary to research a predictive functional control for multivariable systems. McDonnell proposed a predictive functional control method for MIMO system, which has the function of decision logic. However, this method just only be used if the number of input variables are more than the number of output variables, which limits the application of the method. For the variable systems with time delay and external interference, a predictive functional control algorithm by State space equations was derived by Pan². This method can be get the control rule, but derived method is in the form of polynomials and it looks more complicated. Xie³ proposed a new prediction function decoupling control algorithm for variable systems. This algorithm dispersed PFC of variable systems into several PFC of single-variable system, which is greatly simplified the designing parameters and solving algorithm. For the two input and two output variable systems with a inertia first order and pure delay, referring to the Predictor of Smith, considering the impact of pure delay, the
predictive functional control algorithm with distributed decoupling and centralized control was be used by Yue[4]. Wang proposed the variable predictive functional control based on feed-forward compensation decoupling, which separates the system into several multi-input single-output systems. The other input of each system can be taken as a measured disturbance and be applied into temperature control system of a 200MW power plant with steam heat exchanger. Due to MIMO bilinear systems, based on input-output feedback linearization, using hierarchical control strategy, Zhang proposes PFC based on a nonlinear exact feedback decoupling linearization[5].

For the existence of the state delay and the input of control delay, assuming the nominal system without delay analytical, Zhang applied maximum modulus theorem and spectral radius of the matrix properties to give the sufficient conditions of the robust D stability of uncertain state parameter perturbation system[6]. For the multivariable mathematical model with first-order time-delay, Yang uses the single value predictive method and Smith predictor method to get the predictive functional rule with time delay compensation, which was applied to reheat steam temperature system. For the multivariable coupling system with time delay, according to the characteristics of amplitude frequency and phase frequency of the coupling system to decoupling, Zhou designed the predictive functional control rule with the single variable. A partial decoupling design of a state space of PFC to the multivariable system was proposed by Zhang [1], multivariable system can be transformed into the MISO system by the Cramer rule to linearize, and to design the predictive functional controller.

The overall design of controller considered the process state and the dynamic response of output, which has the good performance in terms of tracking the set value and Anti-interference, which is be verified the superiority of the control performance under the condition of model matching and adaptation.

2 Basis Function

Predictive Functional Control takes the structure of control input as a key to influence control performance, which makes it different from traditional predictive control. As a result, PFC regards control input as some pre-selected basis functions with linear combination. The output is a weighted combination of response of the controlled object by the basis functions. For N inputs and N outputs system, control input of \((k+i)T\) is

\[
U(k+i) = \begin{bmatrix}
    u_1(k+i) \\
    u_2(k+i) \\
    \vdots \\
    u_M(k+i)
\end{bmatrix} = \begin{bmatrix}
    \sum_{i=1}^{J} \mu_{ij} f_j(i) \\
    \sum_{i=1}^{J} \mu_{ij} f_j(i) \\
    \vdots \\
    \sum_{i=1}^{J} \mu_{ij} f_j(i)
\end{bmatrix}
\]

(1)
Where $J$ is the number of the basis function, $f_{j}(i)$ is basis function value at the time of $t = (k + i)T$, $i = 0, 1, \ldots, p - 1$; $p$ is prediction time domain; $\mu_{1}, \mu_{2}, \ldots, \mu_{nj}$ is Linear combination coefficients, $n = 1, 2, \ldots, N$.

3 Reference trajectory

For the output of the system can be gradually reaches the set value smoothly in predictive function control, according to the predicted output value and the process output value, we can set a reference trajectory, which is a gradual curve to get the value of the future and is a first order exponential form, so reference trajectory is expressed as follows at the time of $(k + i)T$.

$$
\begin{align*}
Y_{r}(k + i) & = \begin{bmatrix}
    y_{r_{1}}(k + i) \\
    y_{r_{2}}(k + i) \\
    y_{r_{n}}(k + i)
\end{bmatrix} = \begin{bmatrix}
    e_{r_{1}}(k + i) - \lambda_{r_{1}}^{i}[e_{r_{1}}(k) - y_{r_{1}}(k)] \\
    e_{r_{2}}(k + i) - \lambda_{r_{2}}^{i}[e_{r_{2}}(k) - y_{r_{2}}(k)] \\
    e_{r_{n}}(k + i) - \lambda_{r_{n}}^{i}[e_{r_{n}}(k) - y_{r_{n}}(k)]
\end{bmatrix} = \begin{bmatrix}
    e_{r_{1}}(k) \\
    e_{r_{2}}(k) \\
    e_{r_{n}}(k)
\end{bmatrix} = Y_{r}(k) \quad (2)
\end{align*}
$$

4 The output of the predictive model

A predictive model, as the part of PFC, is used to predict the future system output, which only focus on its function rather than its structure. As a result, PFC has many different forms of predictive model. For example, transfer function, state space equation, differential equations, step response, impulse response and so on. In this paper, the state space equations are used as a predictive model.

$$
\begin{align*}
\begin{cases}
X_{u}(k) = A_{u}X_{u}(k - 1) + B_{u}U(k - 1) \\
Y_{u}(k) = C_{u}X_{u}(k)
\end{cases}
\end{align*}
$$

Where $X_{u}(k) = [x_{u_{1}}(k), x_{u_{2}}(k), \ldots, x_{u_{n}}(k)]^{T}$; $U(k) = [u_{1}(k), u_{2}(k), \ldots, u_{n}(k)]^{T}$; $Y_{u}(k) = [y_{u_{1}}(k), y_{u_{2}}(k), \ldots, y_{u_{n}}(k)]^{T}$; $A_{u}, B_{u}, C_{u}$ is the coefficient matrix of the state equation of prediction model.
5 Numerical simulation

As the following 3-input and 3-output liner system:

\[
\begin{bmatrix}
X_2(k) \\
X_3(k) \\
X_4(k)
\end{bmatrix} = \begin{bmatrix}
2 & 5 & 7 \\
3 & 7 & 35 \\
12 & 23 & 44
\end{bmatrix} \begin{bmatrix}
X_2(k-1) \\
X_3(k-1) \\
X_4(k-1)
\end{bmatrix} + \begin{bmatrix}
78 & 34 & 5 \\
44 & 3 & 2 \\
31 & 32 & 56
\end{bmatrix} \begin{bmatrix}
U_1(k-1) \\
U_2(k-1) \\
U_3(k-1)
\end{bmatrix}
\]

(4)

where \( c_1 = 1 \), \( c_2 = 2 \), \( c_3 = 3 \), the basis function is the step function; the sample time \( T_s = 0.01 \); the expected response time \( T_r = 0.5 \); the expected step \( p = 5 \).

When the model is matched, the simulation curves are shown in Fig 1, the solid line is the output curve and the double line is the set value curve.

As can be seen from Fig 1, under the control of MPFC algorithm, numerical stimulation object of the three inputs and three outputs can reach a stable state in 0.7s. If the set value is 1, the output curve can reach a stable state in 0.5s without overshoot. With increasing the set value, the rise time of response curve increases and the overshoot is appeared.
6 Conclusions

The expressions of control of the multivariable predictive function control algorithm can be deduced in this paper. The state space equations are adopted by the controlled object, and then the multivariable system of 3 input and 3 output is simulated by MATLAB. The simulation results show that the control algorithm of MPFC can achieve better control effect and quickly make the system get a stable state, but some overshoot is appeared. The other two curves will be disturbed by changing one of the set values, which shows the selected object have coupling. But MPFC algorithm can be also get better control effect.

Acknowledgments. This work was supported by National Natural Science Foundation of China (nos. 61104062, 61374085 and 61473334), Jiangsu Qing Lan Project, and PAPD.

References