Joint Beamforming and Power Allocation based on CSI

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Abstract. Traditional beamforming and power control algorithms in cognitive radio (CR) are based on the assumption of perfect channel state information (CSI) however, this may lead to performance degradation in realistic systems. In this paper, the problem of joint beamforming and power control is investigated in underlay CR networks with imperfect CSI. Our objective is to maximize the sum utility of secondary users (SUs) under the primary users (PUs) interference power constraints and the transmission power constraint of SUs.

Keywords: Cognitive radio, beamforming, power allocation, imperfect channel state information, Nash equilibrium.

1 Introduction

COGNITIVE radio (CR), as a promising technology to enhancing the utilization efficiency of the scarce radio spectrum, has attracted tremendous interests recently. A key feature of the CR network is to allow a secondary user (SU) to simultaneously share a licensed spectrum as long as the secondary transmission does not interfere with the primary link. As a result, the challenge of the CR network is to protect the primary users (PUs) from harmful interference induced by the SUs as well as to meet the quality of service (QoS) demands of SUs [1].

Cognitive beamforming and power control, as an effective interference suppression technology, has been widely used in CR from different aspects [2]-[4]. All these work are based on the assumption of perfect channel knowledge. However, in practical systems, perfect CSI is difficult to obtain due to the loose cooperation between PUs and SUs, as well as many other factors such as inaccurate channel estimation, limited feedback or lack of channel reciprocity. The worst-case approach has been used to design robust power for SUs in a multiple-input single-output (MISO) CR system [5], [6]. In [5], the software assisted method and a geometric method were considered for single SU and single PU to find suboptimal solution for the certainty and uncertainty models. A bounded region for channel matrices and channel covariance matrices was assumed to be known in [6]. The authors used a type
of ellipsoid uncertainty problem to express the bound channel uncertainty.

## 2 System model and problem formulation

As illustrated in Fig. 1, we consider a cognitive network where a primary network consisting of a PBS and M PUs coexists with a secondary network with a SBS and K SUs. In the secondary network, SUs operate in the frequency band allocated to the PUs, thus the channels between the base stations and users are inherently interference channels. We assume that SBS has \( N_t \) antennas, PBS has a single antenna, and each SU and PU are equipped with only one antenna. For the sake of simplicity, we assume a block fading model for all channels. Let \( h_k \in \mathbb{C}^{1 \times N_t} \), \( k \in [1, K] \), and \( \hat{g}_m \in \mathbb{C}^{1 \times N_t} \), \( m \in [1, M] \) denote the channel gains from the SBS to \( SU_k \) and to \( PU_m \), respectively. Likewise, channel gains between PBS and \( PU_m \) and between PBS and \( SU_k \) are denoted by \( \hat{h}_m \) and \( \hat{g}_k \), respectively. As such, the received signal at \( SU_k \) is represented as

\[
y_k = \sqrt{p_k} h_k f_k s_k + h_k \sum_{i=1, i \neq k}^K \sqrt{p_i} f_i s_i + \hat{g}_k \sum_{m=1}^M \sqrt{p_m} s_m + n_k
\]

(1)

where \( s_k \) and \( s_m \) are the transmitted signals from SBS to \( SU_k \) and that from PBS to \( PU_m \), respectively. Likewise, \( p_k \) and \( p_m \) account for the transmission power of \( SU_k \) and \( PU_m \). \( f_k \) denotes the beamforming vectors of \( SU_k \) and PU. And \( n_k \) is an additive white Gaussian noise with zero mean and variance \( \sigma_n^2 \). The signal-to-interference-plus-noise-ratio (SINR) of the k-th SU is calculated by

\[
SINR_k = \frac{|f_k s_k|^2 p_k}{\sum_{i=1, i \neq k}^K |f_i|^2 p_i + \sum_{m=1}^M |\hat{g}_m|^2 p_m + \sigma_n^2}
\]

(2)

### 3 Experiment Design and Discussion

In this subsection, the optimization problem is solved via a SOCP solution. As is known to all, the zero-forcing (ZF) scheme is a simple and efficient beamforming method which maximizes the sum utility by choosing appropriate \( f_k \). Here, we adopt
the ZF beamforming that transforms the broadcast channel into multi-parallel independent and orthogonal sub-channels. The beamforming vectors are selected to satisfy $h_k f_i = 0, i \neq k$. Suppose that $\hat{F} = [f_1, f_2, \cdots, f_K]$ denotes the beamforming matrix, one easy way to choose the beamforming matrix $F$ that gives the zero interference is as follows.

$$F = H^H \left( HH^H \right)^{-1}$$  \hspace{1cm} (3)

where $H = [h_1, \cdots, h_K]$ denotes the channel matrix and $(\cdot)^H$ denotes the conjugate transpose operation. Here, $F$ is the Moore-Penrose pseudoinverse of the channel matrix $H$. Therefore, the SINR in (2) becomes

$$\text{SINR}_k^* = \frac{\left|h_k f_k \right|^2 p_k}{\sum_{m=1}^M \left|\tilde{g}_m \right|^2 p_m + \sigma_k^2}$$  \hspace{1cm} (4)

Based on (3) and (4), we can perform the power allocation for SUs and rewrite the optimization problem in (5) as

$$\max_{p_k} \sum_{k=1}^K \log \left(1 + k \rho_{m} \Delta f_k \right)$$  \hspace{1cm} s.t. $\sum_{k=1}^K p_k \leq P_T$

$$\sum_{k=1}^K \left|\tilde{g}_m + \Delta m \right|^2 f_k \geq \rho_{m,th},$$  \hspace{1cm} (5)

4 Conclusion

In this paper, the problem of joint beamforming and power control in underlay CR with multiple PUs and multiple SUs was studied. Imperfect CSI between the SBS and PUs was considered. The problem was formulated as non-cooperative game, and then an ellipsoid model was adopted to describe the CSI uncertainty. After making some approximations, the problem was reformulated as a SOCP problem. Simulation results shown that the proposed scheme converges to an equilibrium state. And the sum utilities of SUs were also presented to illustrate the performance of the secondary network under perfect and imperfect CSI.
References

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