# An Analysis on Stability of Competitive Contractual Strategic Alliance Based on the Modified Lotka-Voterra Model

Qing Xuemei[1], Zhang Weiguo[1,2]

[1] School of Economics and Business Administration, Chongqing University, Chongqing 400040, China
[2] College of Economics and Management, Southwest University, Chongqing 400715, China
qingxuemeilx@126.com, wgzhang@cqu.edu.cn

**Abstract:** Original competition, inter-alliance competition and cooperation constitute all together co-opetition relations in competitive contractual strategic alliance, which dictate the stability of the alliance by means of affecting the profits of alliance partners. A modified Lotka-Voterra model is established using system dynamics, due to the similarity of enterprises and alliances to biological systems. According to the analysis of modified Lotka-Voterra model, we find: there exists stable boundary in alliance, the extent of co-opetition within an alliance determines its direction, i.e. differences in the extension of co-opetition result in the differences of alliance revolution; Co-opetition between alliance partners is not always the same, but changing constantly. The system evaluates with its co-opetition changing; Penalty costs and alliance cycle affect the stability of Competitive strategic alliance besides co-opetition.

**Keywords:** Co-opetition, competitive contractual strategic alliance, stability, system dynamics, Lotka-Voterra model

## 1. Introduction

Competitive contractual strategic alliance embodies two entirely different logic principles that contradict each other [1-2], which causes instability of competitive contractual strategic alliance. Usually, instability of such alliance means short and fragile cooperation, and the failure of alliance [3]. However, there are few studies done to examine the stability of competitive contractual strategic alliance. The existing ones still didn't analyze the dynamics and systematics of alliance stability. To make for the deficiencies of existing studies, this paper constructed system cycle diagram using system dynamics, and developed the

dynamics equation, i.e. *Lotka – Voterra* model, for competitive strategic alliance based on contract by applying modified logistic model, so as to discuss the influence of co-opetition on such alliance.

## 2.  Modified Lotka-Voterra Model

Competitive contractual strategic alliance is an open and complicated system. Alliance system's cycle diagram is shown in Fig.1.

Market similarity and contractual control discriminate competitive contractual strategic alliance from other types of alliances. Market similarity dictates that alliance partners are no more than competitors to each other. Contractual control is not only incapable of eliminating competition from partners, but also generates derivations, such as learning competition and bargaining. The former is referred to as original competition, whereas the latter is called inter-alliance competition. The alliance's performance is affected by co-opetition, and dictates the stability of an alliance. Accordingly, the stability mechanism of competitive contractual strategic alliance is shown in Fig.2.
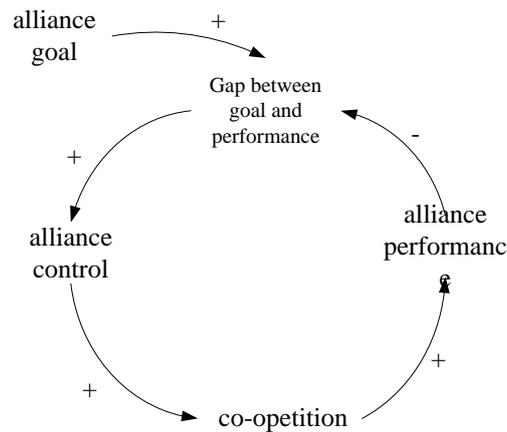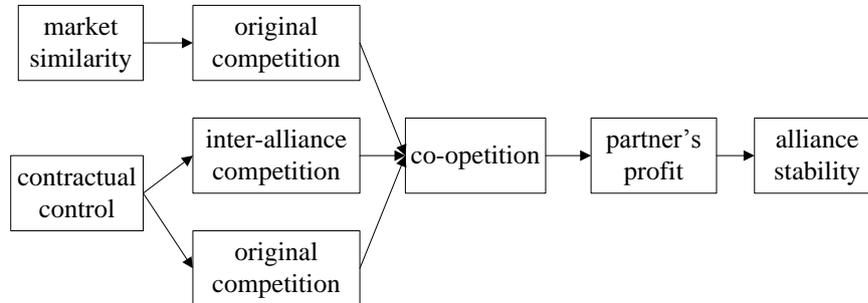


**Fig. 1.** system cycle diagram of competitive contractual strategic alliance

**Fig. 2.** the Stability Mechanism of Competitive contractual strategic alliance[1]

Corresponding intensity of co-opetition is defined as follows: Partner $i$ 's intensity of co-opetition $c_{ij}$ : co-opetition's degree of influence on alliance partner $i$, the variation of partner $i$ 's profit growth rate caused by partner $j$ 's profit change, i.e. : $c_{ij} = \frac{\partial}{\partial R_j}(\frac{dR_i}{dt}) = com_{0i} + com_i^{'} + coo_i$

In the equation, $R_i$ is the profit of partner $i$, $com_{0i}$ is the intensity of original competition, whereas $com_i^{'}$ is the alliance partner $i$ 's intensity of inter-alliance competition; $coo_i$ is the alliance partner $i$ 's intensity of cooperation; $(com_i^{'} + coo_i)$ is the inter-alliance co-opetition effect, namely the alliance effect, $i, j = 1, 2$, $i \neq j$. Notably, the symbol '+' means merging, not adding.

In ecology perspective, enterprises are intelligent living organism with vital signs [4-5]. According to ecological theories, under the condition of partner's individual existence, growth rate of partners' profit follows classical logistic model, whereas alliance's growth rate follows Lotka-Voterra model [6], thus:

$$\frac{dR_i}{dt} = R_i b_i (1 - \frac{R_i}{N_i}) \qquad (1)$$

In the equation, $b_i$ denotes partner $i$ 's internal growth rate, namely the enterprise's growth rate decided by partner's own internal characteristics, and usually $b_i > 0$. $N_i$ denotes the maximum profit partner $i$ can acquire without the variations of time, location and resource.

In equation (1), when $b_i < 0$ and $R_i > N_i$ (alliance's profit surpasses its limit), calculation results in a unreasonable outcome, i.e. $\frac{dR_i}{dt} > 0$. Therefore, classical logistic model has been modified by T.G.Hallam and C.E.Clark as follows:

---

[1] *Alliance* in the Figure is referring to specifically the Competitive contractual strategic alliance.

$$\frac{dR_i}{dt} = R_i(b_i + c_{ii}R_i) \qquad (\ i = 1,2\ ) \qquad\qquad (2)$$

In equation (2), $c_{ii}$ denotes the restriction of necessary investments for partner $i$'s profit growth factors, and $c_{ii} < 0$.

Definition: $u_i = \dfrac{R_i(t)}{N_i}$ denotes partner $i$'s actual market share at time $t$. Obviously, $R_i$ is positively correlated to $u_i$, and in case of partner $i$'s individual existence, $0 \le u_i \le 1$; before alliance, $0 \le u_i < 1$; after, $u_i \ge 0$. Then:

$$\begin{cases} \dfrac{du_1}{dt} = b_1 u_1 (1 - u_1 + \varepsilon_1 u_2) \\ \dfrac{du_2}{dt} = b_2 u_2 (1 - u_2 + \varepsilon_2 u_1) \end{cases} \qquad\qquad (3)$$

This is the modified $Lotka - Voterra$ model based on modified logistic.


## 3.   Model Analysis


### 3.1 Equilibrium

There are four equilibriums in model (4): $P_1\ (0,0)$, $P_2\ (0,1)$, $P_3\ (1,0)$ and $P_4\ (\dfrac{1+\varepsilon_1}{1-\varepsilon_1\varepsilon_2}, \dfrac{1+\varepsilon_2}{1-\varepsilon_1\varepsilon_2})$. According to theory of stability, equilibrium point $P_1$ is unstable under any circumstances; equilibrium $P_2$ gradually stabilizes when $\varepsilon_1 < -1$; equilibrium point $P_3$ gradually stabilizes when $\varepsilon_2 < -1$; equilibrium point $P_4$ stabilizes gradually when $\varepsilon_1 > -1$、 $\varepsilon_2 > -1$ and $\varepsilon_1\varepsilon_2 < 1$.

### 3.3 Stability Boundaries

Positive equilibrium point $P_4$ approaches stability gradually, indicating that competitive contractual strategic alliance is stable. From the stability conditions of equilibrium point $P_4$, the alliance is table when the degree of alliance partner's co-opetition is located in the range of $\varepsilon_1 > -1$、 $\varepsilon_2 > -1$ and $\varepsilon_1\varepsilon_2 < 1$, meaning in this range, although the degree of alliance partners' co-opetition is possibly changed for adaptation, but the alliance remains stable and the system has resistance and recoverability. Outside of this range, the alliance's stability is breached and the alliance cannot continue. Accordingly, the stability boundaries of competitive contractual strategic alliance is obtained and shown in Fig. 3.
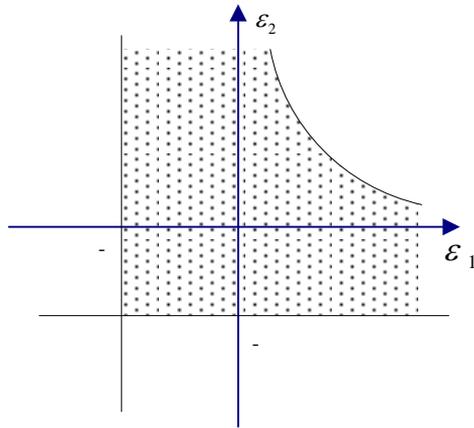
**Fig. 3.** The Stability Boundaries of Competitive contractual strategic alliance

## 4. Further Discussion of the Model

Competitive contractual strategic alliance has three different outcomes, including disintegration, merging and co-existence (passive co-existence and stable co-existence), which correspond to equilibrium points $P_2$、 $P_3$ and $P_4$. Alliance's operation, control and the realization of its target is time consuming. As a result, the actual outcome of alliance evolution is affected by not only the degree of alliance partner's co-opetition, but also alliance cycle and penalty cost.

The stability of competitive contractual strategic alliance is a systematic characteristic manifested under the influence of a series of factors that can be put into competition and cooperation[19] carrying on simultaneously, i.e. co-opetition. After the establishment of alliance, co-opetition between alliance partners is not always be the same, but changing constantly. The continuous motion of co-opetition is the impetus of alliance system's evolution, which place the system in different state of stability.

## 5. Conclusion

Original competition, inter-alliance competition and cooperation constitute all together co-opetition relations in competitive contractual strategic alliance. Co-opetition influences alliance partners by means of affecting their profit, so as to decide the stability of competitive contractual strategic alliance. Under the condition of partner's individual existence, its growth rate follows modified classical logistic

model, whereas alliance's growth rate follows *Lotka – Voterra* model.Through analyzing the modified *Lotka – Voterra* model, it is found:

(1)The differences of co-opetition degree result in different outcomes of alliance system's evolution. There exists stable boundary.

(2) Co-opetition between alliance partners is not always the same, but changing constantly.

(3) Penalty costs and alliance cycle affect the stability of Competitive strategic alliance besides co-opetition, which should be paid more attention.

## References

1. Hill CC., Li H., Davies B, "The paradox of cooperation and competition in strategic alliances: Towards a multi-paradigm   approach ", Management Research News,   vol. 26, no. 1, (2003), pp. 1-20
2. Zineldin M, "Co-opetition the organization of the future", Marketing Intelligence & Planning, vol. 22, no. 6/7, pp. 780-789
3. Kun Zhang, "Study on Strategic Alliance Instability Base on Game Theory", Shanxi University (2010)
4. Zhongzhi Yang, "Introduction to Enterprise Ecology", Science Press, Beijing, (2003)
5. Yuan Lou, Hong Zhao, "The Model of Enterprises Ecosystem and the Analysis of Abiotic Environment Factors", The Journal of Quantitative & Technical Economics, no. 3, (2002), pp. 61-64
6. Linzhou Liu, Bo Wu, Wenxia Sun, "On the Stability of Strategic Alliance of Technological Innovation: Development Model Study", Science & Technology Progress and Policy, no. 3, (2012), pp. 62-64