Two-sided Matching under Uncertain and Incomplete Linguistic Environment

Yue Qi
School of Information Management, Jiangxi University of Finance and Economics, Nanchang, China

Abstract. This paper proposes a two-sided matching method for solving the two-sided matching problem under uncertain and incomplete linguistic environment. In this method, uncertain and incomplete linguistic variable matrices are changed into 2-tuple matrixes. Further, a bi-objective matching model is established. By using the 2-tuple arithmetic mean, the normalization method and the linear weighted method, the multi-objective matching model can be changed into a single-objective matching model. The matching alternative can be determined by solving the matching model. An example is used to illustrate the feasibility of the proposed method.

Keywords: Two-sided Matching; Two-granularity; Uncertain and Incomplete Linguistic Variable; 2-Tuple;

1 Introduction

There are plenty of two-sided matching problems in many fields of real life, such as marriage assignment [1], college admission [2], employee selection [3], and CEOs to companies [4]. At present, the two-sided matching problems with different forms of information have attracted extensively attentions. For example, Gale and Shapley initially investigate the concept, existence, optimality and algorithm of stable matchings [5]. The notions of vonNeumann–Morgenstern stable sets are adopted to determine which matchings are possibly stable when agents are farsighted [6]. Boon and Sierksma match position with player in soccer team formation using linear matching models [7]. Sethuraman et al. focus on the geometric structure of fractional stable matchings in the stable admission problem [8]. Uetake and Watanabe propose an approach to estimate a non-transferable utility in two-sided matching models [9].

The existing studies develop the matching methods, and expand the actual application background. However, due to the complexity of actual problems, the ambiguity of thinking judgment, the preferences provided by agents are in the format of uncertain and incomplete linguistic variables, and the granularity of the linguistic variable set on two sides is also different. The existing studies seldom consider this case. Therefore, how to consider the two-sided matching problem with two-granularity uncertain and incomplete linguistic variables is a valuable research topic. In this paper, a novel two-sided matching method is presented.
2 The Concepts

Definition 1. Let \( S = \{s_0, s_1, \ldots, s_n\} \) be the set of linguistic variables. An uncertain linguistic variable \( \hat{s} \) is expressed in \( \hat{s} = \{s^L, s^{L+1}, \ldots, s^U\} \), where \( s^L, s^{L+1}, \ldots, s^U \in S \), \( L \leq U \). For simplicity, we express \( \hat{s} \) as \( \hat{s} = [s^L, s^U] \).

Definition 2. Let \( \hat{r} = [r^L, r^U] \) be an uncertain linguistic variable, then the probability vector on \( \hat{r} = [r^L, r^U] \) is expressed in \( \hat{p}_r = (p^L_1, \ldots, p^U_1) \), where \( p^L_i = 1/(U - L + 1) \).

Definition 3. Let \( \hat{r} = [r^L, r^U] \) be an uncertain linguistic variable, and \( \hat{p}_r \) be the probability vector on \( \hat{r} \), then by Definitions 2 and 3, the expectation of \( \hat{r} \), i.e., \( E(\hat{r}) \), is calculated by

\[
E(\hat{r}) = \theta \left( \sum_{i=1}^{r} \theta^{-1}(k, 0) \right) \tag{1}
\]

where \( \theta \) is a symbolic translation, and \( \theta^{-1} \) is the inverse function of \( \theta \).

Definition 4. Let \( x = \{(r_{i,j}, \alpha_1), (r_{i,j}, \alpha_2), \ldots, (r_{i,j}, \alpha_f)\} \) be a set of 2-tuples and \( w = \{w_1, w_2, \ldots, w_f\} \) be the associated weights \( (w_i \in [0,1], \sum w_i = 1) \). The extended 2-tuple weighted average is defined as

\[
\tilde{x}^w = \left\{ \begin{array}{ll}
\frac{\theta \left( \sum_{i=1}^{f} \theta^{-1}(r_{i,j}, \alpha_i) w_i \right)}{\sum w_i}, & 0 < \sum w_i \leq 1 \\
\theta(0), & \sum w_i = 0
\end{array} \right. \tag{2}
\]

3 The Problem

This paper considers the two-sided matching problem under uncertain and incomplete linguistic environment. Let \( P = \{P_1, P_2, \ldots, P_n\} \) be the set of agents of side \( P \), where \( P_i \) denotes the \( i \)th agent of side \( P \); \( Q = \{Q_1, Q_2, \ldots, Q_m\} \) be the set of agents of side \( Q \), where \( Q_i \) denotes the \( j \)th agent of side \( Q \). Let \( S_P = \{s^P_0, s^P_1, \ldots, s^P_n, \theta\} \) be the extended set of satisfaction linguistic variables of side \( P \); \( U_P = [u^P_0]_{m \times n} \) be the satisfaction matrix from side \( P \) to \( Q \), \( u^P_{i,j} = [u^P_{i,j}^L, u^P_{i,j}^U] \), \( u^P_{i,j}^L, u^P_{i,j}^U \in S_P \), \( u^P_{i,j}^L \leq u^P_{i,j}^U \). Let \( S_Q = \{s^Q_0, s^Q_1, \ldots, s^Q_n\} \) be the extended set of satisfaction linguistic variables of side \( Q \); \( U_Q = [u^Q_0]_{m \times n} \) be the satisfaction uncertain linguistic variable matrix from side \( Q \) to \( P \), \( u^Q_{i,j} = [u^Q_{i,j}^L, u^Q_{i,j}^U] \), \( u^Q_{i,j}^L, u^Q_{i,j}^U \in S_Q \), \( u^Q_{i,j}^L \leq u^Q_{i,j}^U \).
The problem is how to obtain the matching alternative based on two-granularity uncertain and incomplete linguistic variable matrixes $U_p = [\hat{u}_{ij}]_{m \times n}$ and $U_q = [\hat{u}_{ij}]_{m \times n}$. 

4 The Method

The uncertain and incomplete linguistic variable matrixes $U_p = [\hat{u}_{ij}]_{m \times n}$ and $U_q = [\hat{u}_{ij}]_{m \times n}$ are firstly changed into 2-tuple matrixes $\hat{P} = [(a_{ij}^p, \alpha_{ij}^p)]_{m \times n}$ and $\hat{Q} = [(a_{ij}^q, \alpha_{ij}^q)]_{m \times n}$ by Eq. (1).

Next, we consider constructing a matching model under the matching constraints. On the one hand, according to the characteristics of linguistic variable and 2-tuple, we know that the greater $(a_{ij}^p, \alpha_{ij}^p)$ or $(a_{ij}^q, \alpha_{ij}^q)$ is, the greater the satisfaction degree is.

On the other hand, the matching constraints can be interpreted as $\sum_{i=1}^{m} x_{ij} \leq 1$ and $\sum_{j=1}^{n} x_{ij} \leq 1$, where $x_{ij} = \begin{cases} 1, & \mu(P_i) = Q_j \\ 0, & \mu(P_i) \neq Q_j \end{cases}$. Furthermore, considering that the status of each agent of one side is usually the same, the following bi-objective matching model (3) can be established by using Eq. (2) and the 2-tuple arithmetic mean [10]:

$$\text{max} \quad Z_{(p)} = \theta \left( \frac{1}{m} \sum_{i=1}^{m} \sum_{j=1}^{n} \theta^{-1}(a_{ij}^p, \alpha_{ij}^p) x_{ij} \right)$$

$$\text{max} \quad Z_{(q)} = \theta \left( \frac{1}{n} \sum_{j=1}^{n} \sum_{i=1}^{m} \theta^{-1}(a_{ij}^q, \alpha_{ij}^q) x_{ij} \right)$$

s.t. $\sum_{j=1}^{n} x_{ij} \leq 1, \quad i = 1, 2, \ldots, m \quad (3c)$

$\sum_{i=1}^{m} x_{ij} \leq 1, \quad j = 1, 2, \ldots, n \quad (3d)$

$x_{ij} \in \{0, 1\}, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n \quad (3e)$

Considering the characteristics of symbolic translation $\theta$, the optimal solution of the following matching model (4) is that of model (3):

$$\text{max} \quad Z_{(p)} = \frac{1}{m} \sum_{i=1}^{m} \sum_{j=1}^{n} \theta^{-1}(a_{ij}^p, \alpha_{ij}^p) x_{ij} \quad (4a)$$

$$\text{max} \quad Z_{(q)} = \frac{1}{n} \sum_{j=1}^{n} \sum_{i=1}^{m} \theta^{-1}(a_{ij}^q, \alpha_{ij}^q) x_{ij} \quad (4b)$$

subject to Eqs. (3c)-(3e).

To solve model (6), due to $p \neq q$, Eqs. (4a) and (4b) should be normalized firstly. Let $w_p$ and $w_q$ be the weight of $Z_{(p)}$ and $Z_{(q)}$, such that $w_p, w_q \in [0, 1]$,
w_j + w_q = 1, then model (4) can be changed into the following single-objective matching model (5):

\[
\max Z = \frac{1}{mn} \sum_{m=1}^{m} \sum_{n=1}^{n} (w_j \theta^{-1}(u^{\theta}_j, \alpha^{\theta}_j) + w_q \theta^{-1}(u^{\theta}_q, \alpha^{\theta}_q)) x_{ij}
\]

subject to Eqs. (3c)-(3e). Therefore, the matching alternative can be determined by solving model (5).

4 Conclusion

A matching method for solving the two-sided matching problem under uncertain and incomplete linguistic environment is proposed. Comparing with the existing methods, the proposed method has distinct characteristics as discussed below.

Firstly, the related concepts of uncertain linguistic variable and 2-tuple are given. It is a beneficial supplement of theory of linguistic variable. Secondly, a bi-objective matching model is constructed. By solving it, it could be easier to obtain the reasonable stable matching alternative. Finally, the proposed method is theoretically sound and computationally simple and can be adopted for practical use.

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