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Approximation of High Order Characteristic Equation using Discrete Haar Transforms

Joon-Hoon Park

Korea National University of Transportation, Korea
jhpark@ut.ac.kr

Abstract. An algorithm for approximation high order characteristic equation by low order characteristic function using discrete Haar transform is presented in this paper. The characteristic equation often contains less significant poles that have little effect on the system response. Haar function set forms a complete set of orthogonal rectangular functions similar in several respects of the Walsh functions. The method adopted in this paper is that of system approximation using discrete Haar transform. This approach provides a more efficient and convenient method for the system order reduction.

Keywords: characteristic equation, approximation high order system, discrete Haar functions, transforms

1 Introduction

In control systems, the characteristic equation does an important role in system analysis and design. They can be defined with respect to differential equations, state equations, or transfer functions.[1] For examples, the stability of linear time invariant systems can be determined by checking on the location of the roots of the characteristic equation of the system. Haar function and transform that are introduced in this paper are useful to approximation of characteristic equation for system analysis and reduction. The Haar functions form an orthogonal and orthonormal system of periodic square waves. If we consider the time base to be defined as $0 \leq t \leq 1$ then, the Haar functions is described as follows.[2]

$$Har(0,t) = 1 \quad for\ \ 0 \leq t \leq 1 \tag{1.1}$$

$$Har(1,t) = \begin{cases} 1 & for\ \ 0 \leq t \leq \frac{1}{2} \\ -1 & for\ \ \frac{1}{2} \leq t \leq 1 \end{cases} \tag{1.2}$$

$\vdots$

$Har(2^p + n, t) =$

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$$\begin{cases} \sqrt{2^p} \quad for \quad \frac{n}{2^p} \le t \le \frac{n+\frac{1}{2}}{2^p} \\ -\sqrt{2^p} \quad for \quad (n+\frac{1}{2})/2^p \le t \le (n+1)/2^p \\ 0 \quad for \quad elsewhere \end{cases} \tag{1.3}$$

where $p = 0, 1, 2 \dots . \log_2 \frac{m}{2},\ n = 0, 1 \dots 2^p - 1$

2 Discrete Haar Transforms

A function *f(t)* is absolutely integral on then it can be expanded as an infinite series in term of Haar functions.

$$f(t) = f_0 h_0(t) + f_1 h_1(t) + f_2 h_2(t) + \cdots$$
$$= \sum_{i=0}^{\infty} f_i h_i(t) \tag{2.1}$$

Where f_i is the *i*th sequentially ordered coefficient of the Haar functions expansion of function *f(t)* and h_i is the *i*th ordered Haar functions. The coefficient of the Haar functions expansion is given as equation (2.2).

$$f_i = \int_0^1 f(t) h_i(t) dt \tag{2.2}$$

We can get the approximation of *f(t)* using Haar transform and its matrix expression.

$$f(t) = \sum_{i=0}^{n-1} f_i h_i(t) = F_n^T Har_n(t) \tag{2.3}$$

where F_n is coefficient vector of *f(t)* and $H_n(t)$ is its Haar functions vector. T denotes transposition. For example, if *f(t)=t*, *f(t)* can be transformed and approximated using Haar functions and the result is shown in figure 1.[3]

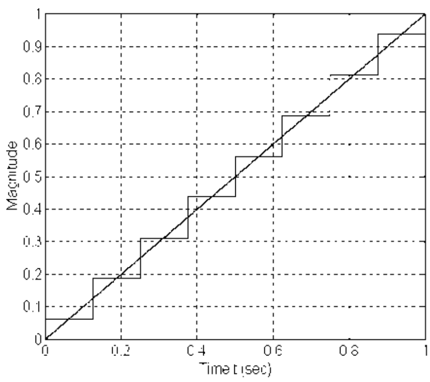

Fig. 1. Haar transform of *f(t)=t* with *n=8*

Thus discrete Haar transforms can be written as follows. In equation (2.4), f_i^* is discrete sampling results of *f(t)* and F_i is coefficient of discrete Haar transforms.[4]

$$f_i^* = \sum_{i=0}^{m-1} F_i Har(n,i) \tag{2.4}$$

$$F_i = \frac{1}{m}\sum_{n=0}^{m-1} Har(i,n)\, f_n^* \tag{2.5}$$

$$f^* = [f_0^* \ f_1^* \ f_2^* \ \ldots \ f_{m-1}^*]^T, \quad \mathrm{F} = [F_0 \ F_1 \ F_2 \ \ldots \ F_{m-1}]^T$$

$$Har_i = [Har_{i0} \ Har_{i1} \ Har_{i2} \ \ldots \ Har_{i(m-1)}]^T \tag{2.6}$$

And we can obtain matrix expression of discrete Haar transforms by equation (2.7) and (2.8).[5]

$$f^* = Har^T F \tag{2.7}$$

$$F = \frac{1}{m} Har f^* \tag{2.8}$$

where *Har* is *m*x*m* orthogonal functions matrix.

3 Characteristic Equation Approximation using Discrete Haar Transforms

The transfer function of the system is described by equation (3.1) and the characteristic equation is obtained by equating the denominator polynomial of the transfer function to zero.

$$\mathrm{G(s)} = \mathrm{K}\frac{1+b_1 s+b_2 s^2+\cdots+b_m s^m}{1+a_1 s+a_2 s^2+\cdots+a_n s^n} \tag{3.1}$$

Given a high order characteristic equation $C_H(s)$, we can find a low order characteristic equation $C_L(s)$ as an approximation. A method of approximating high order system by low order system is based on one in the sense that the frequency responses of two systems are similar. Let the characteristic equation of high order system as equation (3.2) and the approximating low order system as equation (3.3).[6]

$$C_H(\mathrm{s}) = K(1 + a_1 s + a_2 s^2 + \cdots + a_n s^n) \tag{3.2}$$

$$C_L(\mathrm{s}) = K(1 + d_1 + d_2 s^2 + \cdots + d_p s^p) \tag{3.3}$$

where $n \geq m$, $n \geq p \geq q$. In this case, the zero frequency gain K of the two transfer functions is the same. And $s=jw$ is applied to above equations. Thus we can obtain equation (3.4) and (3.5) respectively.

$$C_H(jw) = k(A(w) + jwB(w)) \tag{3.4}$$

$$C_L(jw) = k(C(w) + jw\mathrm{D}(w)) \tag{3.5}$$

Now, we can apply discrete Haar transforms, the coefficients of low order characteristic equation are defined using equation (3.6) to (3.7) conveniently. The coefficients can be obtained from the high order characteristic equation.

$$C(\mathrm{w}) = \frac{1}{n}\sum_{i=0}^{n-1} Har_i(w)c_i^*,\ c_i^* = \int_0^1 A(w)Har_i(t)dt \tag{3.6}$$

$$\mathrm{D}(\mathrm{w}) = \frac{1}{n}\sum_{i=0}^{n-1} Har_i(w)d_i^*,\ d_i^* = \int_0^1 \mathrm{B}(w)Har_i(t)dt \tag{3.7}$$

This method is useful to convert high order characteristic equation into low order characteristic equation.

4 Simulation

Consider a control system that has forward path transfer function and feedback control path with gain 3. The system can be described by equation (4.1).

$$\mathrm{G(s)} = \frac{2}{s(s^2+2s+3)}\ , H(s) = 3 \tag{4.1}$$

Thus, the characteristic equation of the high order system is written as,

$$C_H(\mathrm{s}) = s^3 + 2s^2 + 3s + 6 \tag{4.2}$$

From equation (4.2), the low order system is the second order system and the characteristic equation $C_L(s)$ described in equation (4.3) is obtained using a numerical method. We can compare the results of proposed method in this paper with the results of numerical method.

$$C_L(\mathrm{s}) = s^2 + s + 1.73 \tag{4.3}$$

Now we apply the discrete Haar transforms that is suggested in this paper to get the low order characteristic equation $C_L(s)$. Then discrete coefficients with $n=8$ is obtained as table 1.

Table 1. Discrete coefficients with n=8

Discrete coefficients	Results
d_0^*	0.0491
d_1^*	0.0481

d_2^*	0.0628
d_3^*	-0.0003
d_4^*	0.0891
d_5^*	-0.0035
d_6^*	-0.0001
d_7^*	-0.0001

5 Conclusions

The characteristic equation is important to system analysis, especially to decide the stability of the system. However, system order can be reduced by eliminating of an insignificant pole or root from the characteristic equation. In this paper, the discrete Haar transforms are used as an approximating set of system order reduction of the characteristic equation and transfer function. The proposed result is similar to the original third order system and the method for approximating characteristic equation is simple and useful.

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