

Applications of Hamming Codes to Public Safety Mobile Communication Systems

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Abstract. In this paper, we introduce Hamming codes in modern public safety mobile communication systems such as digital mobile radio (DMR) and association of public safety communications officials-project-25 (APCO-25) systems with performance analysis. Although Hamming codes are the oldest error correcting codes, they are still widely used in the field of mobile communication systems because of the excellent distance property, the good algebraic structure, and the ease of implementation.

Keywords: Hamming code, DMR, APCO-25, public safety, mobile communication systems

1 Introduction

Hamming codes are the oldest error correcting codes proposed by Hamming in 1950 [1]. The codes are still widely used in many modern mobile communication systems such as digital mobile radio (DMR) and association of public safety communications officials-project-25 (APCO-25) systems [2]-[3]. DMR and APCO-25 systems are developed by Telecommunication Industry Association (TIA) and European Telecommunications Standards Institute (ETSI), respectively. In this paper we show that (n, k) Hamming codes in DMR and APCO-25 systems are derived from the primitive polynomial of degree r , where $n = 2^r - 1$ and $k = 2^r - r - 1$. Deleting the first l message bits of (n, k) Hamming codes, we get $(n - l, k - l)$ shortened Hamming codes suitable for the frame structure of the systems. Simulation results for Hamming codes in the systems using syndrome decoding are also presented. For the better performance enhancement of Hamming codes, we can apply maximum likelihood decoding (MLD) using a trellis presented by Wolf, which is very practical in decoding high-rate codes since the complexity of the algorithm is upper-bounded by a function of the number of parity symbols [4]-[5]. We can also use a sub-optimum decoding algorithm proposed by Chase, but which does not always achieve maximum likelihood decoding [6].

2 Hamming codes in DMR system

In DMR system, Hamming codes with various code rates of $(n, k, d) = (7, 4, 3), (15, 11, 3), (16, 11, 4), (13, 9, 3)$, where n is the length of code word, k is the number of information bits, and d is the minimum distance of the code. Hamming codes in DMR system are cyclic codes generated by the primitive polynomials of degree 3, 4, and 5 as described in table 1.

Table 1. Hamming codes in DMR system

Hamming code	Generator polynomial
(7,4,3)	$G(x) = x^3 + x + 1$
(15,11,3), (16,11,4)	$G(x) = x^4 + x + 1$
(13,9,3)	$G(x) = x^5 + x^2 + 1$

Use the following generator matrix,

$$G = \begin{bmatrix} x^{n-1} + r_{k-1}(x) \\ x^{n-2} + r_{k-2}(x) \\ \vdots \\ x^{n-k} + r_0(x) \end{bmatrix} = [I_k | P] \quad (1)$$

we encode the Hamming codes in systematic form, where $r_{k-i}(x) = x^{n-i} \bmod G(x)$, $1 \leq i \leq k$ [7]. The generator and parity check matrix of (n, k, d) Hamming code are $G = [I_k | P]$ and $H = [P^T | I_{n-k}]$. The parity matrix P can also be viewed as

$$P = \begin{bmatrix} \alpha^{n-1} \\ \alpha^{n-2} \\ \vdots \\ \alpha^{n-k} \end{bmatrix}$$

where α is the primitive element of $G(x)$. From (15,11,3) Hamming code we obtain the extended (16,11,4) Hamming code by adding one parity-check bit. Deleting the first 18 information bits of (31,26,3) Hamming code generated by $G(x) = x^5 + x^2 + 1$, we obtain (13,9,3) shortened Hamming code. Figure 1 shows the performance of the Hamming codes over AWGN (Additive White Gaussian Noise) channel in DMR system using syndrome decoding.

3 Hamming codes in APCO-25 system

In APCO-25 system, two Hamming codes, (15,11,3) and (10,6,3), are used generated by $G(x) = x^4 + x + 1$. Deleting the first 5 information bits of the (15,11,3) code, we obtain the shortened (10,6,3) Hamming code. Figure 2 shows the performance of the codes in APCO-25 system.

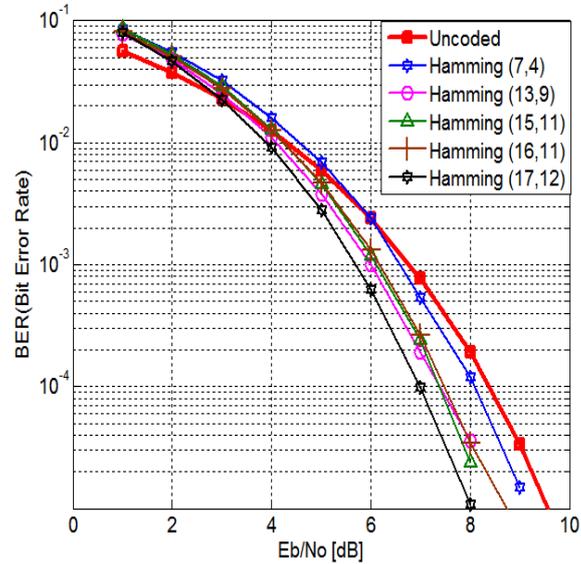


Fig.1.Performance of Hamming codes in DMR system.

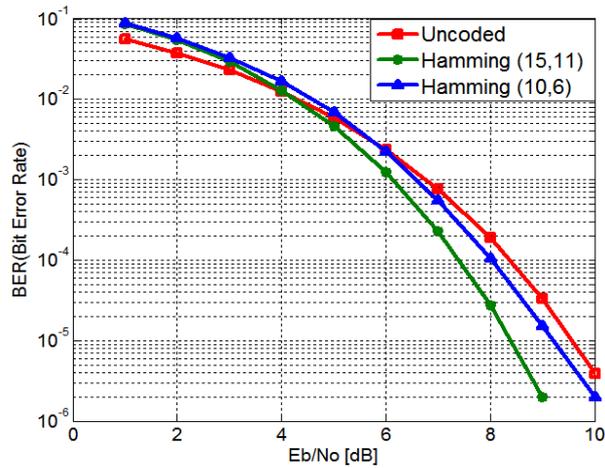


Fig. 2.Performance of Hamming codes in APCO-25 system.

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