Analytical Evaluation of MDPSK and MPSK Modulation Techniques over Nakagami Fading Channels

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Abstract. To design the wireless communication channel, it is required to know the statistical model of channel accurately. Nakagami distribution is extremely convenient tool for analyzing the performance of wireless fading channels. In this paper, a unified analytical framework is presented to obtain the closed-form solutions for calculating the average symbol error rates of MDPSK and coherent MPSK modulation, with or without diversity reception over Nakagami-\(m\) fading channels. Result shows that an error rate of the fading channel typically depends on Nakagami parameters (\(m\)) and the space diversity (\(N\)). The fading effect of channel is inversely proportional to \(m\) and \(N\) that represents, for high value of \(m\), the channel acts like AWGN transmission and the low value of \(N\) creates high fading environments. Also this paper portrays a comparison between MPSK and MDPSK modulation formats. The result proves that MPSK demonstrates better performance compared to MDPSK over any fading and non-fading conditions.

Keywords: Fading channels, M-ary Differential Phase-Shift Keying (MDPSK), M-ary Phase-Shift Keying (MPSK), Nakagami-\(m\), Probability Density Function (PDF), Symbol Error Rate (SER)

1 Introduction

In mobile communication systems, the propagation of signal is characterized by three Non Line Of Sight (NLOS) conditions. These are reflection, diffraction, and scattering which roughly illustrate the radio propagation by three other phenomena like path loss, shadowing and multipath fading. Shadowing and multipath fading are known as long term and short term fading respectively. These two phenomena can be statistically described by the fading models [1]. The analytical methods of this paper are applied for calculating the SER of both a coherent M-ary phase shift keying (MPSK) system and an M-ary differential phase shift keying (MDPSK) over slow and flat Nakagami-\(m\) fading channel. Nakagami distribution sometimes is considered as good in term of flexibility and accuracy than Rayleigh, Rician or log-normal distributions [2].

Several statistical distributions have been proposed by the researchers for channel modeling of fading envelope under short and long term fading conditions. Short-term fading models include the Rayleigh, Weibull, Rice, and Nakagami distributions [1].
Turin [3] proposed one model that associates each path with a set of three random variables like the strength, the modulation delay and the carrier phase shift [2]. When the delay is unknown, the fading is followed as Nakagami distribution, which seems to perform better than Rice distribution [2]. Also Fotohabadi et. al. [7] showed that the consumed power in Nakagami sub-channel is less than that in Rayleigh sub-channel. George et. al. [1] investigated the suitability of Nakagami distribution to approximate the lognormal distribution. Juang et. al. [8] presented a method for parameter selecting of a finite-state Markov chain to match with Nakagami-m fading channel.

In this paper, the performances of different modulation techniques are simulated over Nakagami-m fading channel.

2  Statistical Characteristics of Nakagami-m fading channel

The PDF of a received signal under Nakagami fading with parameters \( m \) and \( \Omega \) is given by:

\[
P(a) = \frac{2}{\mathcal{G}(m)} \left( \frac{m}{\Omega} \right)^m a^{2m-1} e^{-\frac{m}{\Omega} a^2}, \quad a \geq 0
\]

where \( \mathcal{G}(m) \) is the Gamma function, \( a \) is strength of received signal, \( \Omega = E[a^2] \) and \( m \geq \frac{1}{2} \). The Nakagami distribution becomes Rayleigh if the value of \( m=1 \). A Nakagami distribution \((m, \Omega)\) can approximate a Rice distribution [2] with \( m=1 \). For a given distribution, we shall define the amount of fading (AF) is \( AF = \frac{\text{var} \{a^2\}}{E[a^2]^2} \). For the Nakagami distribution, \( AF = \frac{m}{m+1} \) and \( 0 \leq AF \leq 2 \) [2]. In particular case when \( m = \infty \), then \( AF = 0 \) which corresponds to a situation of “no fading”; for \( m = 1 \), \( AF = 1 \) which corresponds to “Rayleigh fading” and for \( m = 1/2 \), \( AF = 2 \), which is the severest fading assumed by Nakagami fading.

3  SER derivation of MPSK over Nakagami-m fading channels

It can be assumed that there are \( N \) diversity channels carrying the same information bearing signal. Each channel is modeled as frequency non-selective slowly Nakagami-m fading channel corrupted by additive white Gaussian noise (AWGN) process. In fading, instantaneous SNR \( \{y_k\} \) becomes a random variable and assume that the channel is identical, the PDF of \( y_k \) at the output of the maximum ratio combiner (MRC) may be written as [4],

\[
P_e(y_k) = \left( \frac{mN}{T} \right)^{mN} \left( \frac{y_k^{mN}}{\mathcal{G}(mN)} \right)^m e^{-\frac{mN}{T} y_k^2} \sqrt{\frac{mN}{\mathcal{G}(mN)}}
\]

(1)
where, \( G() \) is the gamma function, \( m \) is the Nakagami fading parameter, which is assumed to be identical for all channels and \( \Gamma \) is the average SNR associated with each symbol, which is related to \( \bar{P}_b \) as \( \bar{P} = \bar{N} \bar{P}_b \). If the PDF of \((\gamma_b)\) is known, then the average SER \( P_{ser} \) in fading can be calculated by the following equation (2):

\[
P_{ser} = \int_0^\infty P_b(\gamma_b)G(\gamma_b)\,d\gamma_b
\]  

(2)

In here, \( P_b(\gamma_b) \) is the conditional probability of symbol error over AWGN channel. The SER of M-PSK is given in the exponential form and can be written as [5],

\[
P_b(\gamma_b) = \frac{1}{\pi} \int_{-\infty}^{\infty} e^{\gamma_b \sin^2 \left( \frac{\pi}{2} \right) \sec^2 \theta} \,d\theta
\]

(3)

Equation (4) represents the integral relationship explained in [6].

\[
\int_0^\infty x^{n-1}e^{-ax}\,dx = G(n)\frac{1}{a^n}
\]

(4)

By substituting equations (1) and (3) in equation (2), and using the relationship of equation (4), the average SER over Nakagami fading channel with \( N \) order diversity is,

\[
P_{ser}(\gamma_b) = \frac{1}{\pi} \int_{-\infty}^{\infty} \left[ 1 + \frac{\sin^2 \left( \frac{\pi}{2} \right) \sec^2 \theta}{mn} \right]^{nN} \,d\theta
\]

(5)

Equation (5) represents the average SER of coherent M-PSK over Nakagami-\( m \) fading channels. If \( m = \infty \), equation (5) represents the equation (3) which is the SER of M-PSK over AWGN channel.

4 SER Derivation of MDPSK over Nakagami-\( m \) Fading Channels

It is shown in [5] that the probability of symbol error of M-DPSK over AWGN channel is given as

\[
P_b(\gamma_b) = \frac{\sin \left( \frac{\pi}{2m} \right)}{2\pi} \int_{-\infty}^{\infty} \frac{\exp \left[ -\gamma_b (1 - \cos \left( \frac{\pi}{2m} \right)) \cos \theta \right]}{(1 - \cos \left( \frac{\pi}{2m} \right)) \cos \theta} \,d\theta
\]

(6)

Substituting the values from (6) and (1) into equation (2), and using the relation of equation (4), the average symbol error probability of MDPSK over Nakagami fading channel is,

\[
P_{ser}(\gamma_b) = \frac{\sin \left( \frac{\pi}{2m} \right)}{2\pi} \int_{-\infty}^{\infty} \left[ 1 + \frac{\gamma_b (1 - \cos \left( \frac{\pi}{2m} \right)) \cos \theta}{mn} \right]^{nN} \,d\theta
\]

(7)
5 Evaluation of MPSK over Nakagami-m Fading Channel

Equation (5) shows the exact numerical represent of SER of M-PSK modulation technique over Nakagami-m fading channel. Here, $\Gamma$ represents the average signal to noise ratio in dB and $N$ shows the number of available channels. So SER over Nakagami-m fading channel can be plotted by varying the channel numbers ($N$) and the number of symbols $M$ for PSK modulation. If the value of $m = 1$, it shows Rayleigh fading channel and it represents AWGN channel for $m >> 1$. Interestingly it can be shown that the increasing value of $m$ coverts the channel from Nakagami to AWGN. Figure 1 depicts the average symbol error rate of BPSK ($M = 2$) modulation. In this figure, number of channel is one ($N = 1$) and $m$ is varied from 0.5 to $10^6$ (theoretically infinity). From this figure, it is easily showed that when $m = 0.5$, its performance is worse and when $m$ is gradually increased then the channel converts to non fading channel like AWGN. But, here the overall performance of fading channel is not so good because it is plotted for $N = 1$ (no diversity). Figure 2 shows the symbol error rate for BPSK modulation with multiple channel numbers ($N = 2, 12$). It is plotted for same conditions as figure 1 except diversity. Here diversity is applied ($N > 1$) and it shows good performance compared to figure 1. If the fading effects are gradually reduced then channel goes close to AWGN channel. For example from these two figures, it can be shown that for Nakagami channel ($m = 2$) SER is almost near to $10^{-3}$ at 15 dB SNR for without diversity condition but when diversity is applied then SER is reduced and almost equals to $10^{-5}$.

Figure 3 is simulated for different symbol values of PSK ($M = 4, 16, 64$) modulation technique including diversity and without diversity. For severe fading channel ($m = 0.5$), figure 3 shows that, when number of symbol is low ($M = 4$), the performance is high. But as the fading parameter $m$ improves from 0.5 to 1, there is significant improvement of response curve shown in figure 3 for without diversity conditions. Similarly, from figure 3, low symbol value of PSK shows good performance because if the phase variation of carrier signal is low then the phases of carrier are less corrupted by channel. Comparing the different curves in figure 3, it can be said that the vulnerability increases with $M$. For example, at a given SNR, the SER is raised least for $M = 4$ and greatest for $M = 64$. 

Fig. 1. SER of BPSK for $N = 1$ (without diversity) over different fading channels

Fig. 2. SER of BPSK for $N = 2, 12$ (with diversity) over different channels

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Evaluation of MDPSK over Nakagami-\(m\) Fading Channel

Equation (7) shows the exact numerical presentation of symbol error rate for MDPSK over Nakagami-\(m\) fading channel. Figure 4 is simulated for DBPSK over different fading channels without diversity condition but diversity is applied for only Nakagami parameter \(m = 0.5\). From this figure, it can be said that when diversity is applied then the fading channels show good result compared to without diversity condition. Figure 5 shows the performance for \(M = 4, 16, 64\) DPSK in various symbols (different \(M\)) including with and without diversity. For fixed value of fading parameter \(m = 5\), \(M = 4\) DPSK shows good result compared to other values of \(M\). Now if we compare the result between MPSK and MDPSK over Nakagami fading channel then MPSK is an attractive communication technique and shows very good performance than MDPSK modulation technique. Figure 6 shows such simulation result for \(M = 4, 16, 64\) DPSK and PSK over Nakagami fading channel. From the above simulation curves, it can be written that there should be a tradeoff between fading parameter \(m\) and diversity \(N\). If channel behaves like strong fading effects (for low value of \(m\)) then it is wise to increase diversity. Similarly if there is an AWGN channel (\(m = \infty\)) then it is not
necessary to apply diversity. Therefore, in nutshell, number of symbols \((M)\) and SNR are significant parameters to design efficient communication system. The increasing tendency of \(M\) consumes large SNR at the given error rate. It is not always possible to transmit data with high rate. Depending on the channel response, designers will assign the symbol rate by observing the error responses.

### 7 Conclusions

We outline analytical solutions for computing average SER of MDPSK and MPSK over Nakagami fading channel. These are simulated and compared for various values of fading parameter \(m\). In this paper, PSK modulation is widely analyzed for different fading channels using space diversity. For that reason engineers can get useful information to evaluate and design the radio communication link over Nakagami fading channel.

### References