Abstract. In this paper, an improved fusion algorithm of evidence theory is presented. Firstly, it uses the evidence distance to obtain the corresponding of evidence conflict, then obtains the evidence credibility by using the entropy of evidence conflict and makes use of the evidence credibility to redistribute the basic probability distribution function. Finally, it gets the fusion result by the ordinary combination formula. The simulation results prove that the algorithm can fuse the evidence with serious conflict effectively.

Keywords: Evidence Theory; Evidence Fusion; Evidence Distance

1 Introduction

As an effective fusion, Dempster-Shafer evidence theory\(^1\)\(^-\)\(^2\) has been widely used method in multi-sensor data fusion. Comparing with the traditional probability fusion method, it has a better capacity of grasping unknown and uncertain problem when it is regarded as an uncertainty method. In addition, it provides an important composite formula which can fuse evidence of different sensors. However, it also exists many deficiencies. Firstly, it can not resolve the problem of evidence conflict; Secondly, the small disturbance of the basic probability distribution function brings a drastic changes in fusion result. Thoroughly considering the importance of evidence in the fusion process, Reference[3] introduces weight coefficient and redistribution of conflict probability. It can improve the rationality and reliability of fusion result, but it does not work well when there are conflicts between the evidences. In this paper, a new fusion rule is presented, which is better than reference [3].

2 Evidence theory and resultant formula

Dempster-Shafer evidence theory defines a space \(\Theta\), which is called identified framework, being made up of some mute and exhaustive elements. It defines mapping \(m : 2^{\Theta} \rightarrow [0, 1]\) as the basic probability distribution function for all the topics of \(A\), \(m\) satisfies that:

1. \(m(\emptyset) = 0\);
2. \(0 \leq m(A) \leq 1\);
3. \(\sum_{A \subseteq \Theta} m(A) = 1\).
If \( A \subseteq \theta \) and \( m(A) \neq 0 \), it calls \( A \) as the focus of \( m \).

Dempster-Shafer evidence theory provides the combine formula as follow:

\[
\begin{align*}
(m(\phi) &= 0 \\
 m(A) &= \frac{\sum_{i=1}^{n} m_i(A_i) \prod_{i=1}^{n} m_i(A_i)}{1 - \sum_{i=1}^{n} m_i(A_i)} \quad (1)
\end{align*}
\]

The value of \( k \) represents the conflicts of all the evidences, and \( 1 - k \) is called normalization factor which can prevent setting non-zero probability to empty set when it combines the evidences.

3 An improved fusion algorithm of evidence theory

Firstly, it uses the evidence distance to obtain the corresponding of evidence conflict, and then gets the credibility by using the entropy of evidence conflict, and next makes use of the credibility of the evidence to redistribute the basic probability distribution function. Finally, it gets the fusion result by the ordinary combination formula.

Suppose \( \theta \) is an identifiable framework which contains different topics of \( n \), \( m_i \) and \( m_j \) are two evidences of \( \theta \). It lets \( A = \{ A_1, A_2, \cdots A_r \} \), and the weight vector is:

\[ W_j = \{ \omega_1, \omega_2, \cdots, \omega_n \} \quad i = 1, 2 \cdots n \]

Suppose the basic probability distribution can be shown as follow:

\[ m_j = [ m(A_1), m(A_2), \cdots, m(A_r) ] \quad (2) \]

Where, \( m_i(A_k) \geq 0, k = 1, 2, \cdots , r \), \( \sum_{k=1}^{r} m_i(A_k) = 1 \),

The evidence distance between \( m_i \) and \( m_j \) is shown as follow:

\[
d(m_i, m_j) = \frac{1}{2} \left( m_i - m_j \right) D \left( m_i - m_j \right) \quad (3)
\]

Where, \( D \) is a \( r \times r \) matrix, the element of \( D \) can be shown as follow:

\[
D(A_i, A_p) = \begin{cases} 
\frac{A_i \cap A_p}{A_i \cup A_p} & l, p = 1, 2 \cdots r
\end{cases} \quad (4)
\]

It can obtain the concrete calculation method from formula (4) and (5):
\[
\|m\| = (m, m), \text{ the inner product between } m_i \text{ and } m_j \text{ is:}
\]
\[
(m_i, m_j) = \sum_{j=1}^{r} \sum_{p=1}^{r} m_i(A_j) m_j(A_p) \frac{|A_j \cap A_p|}{|A_j \cup A_p|}
\]

The distance matrix can be presented as follow:
\[
D(m_i, m_j) = \begin{bmatrix}
0 & d_{i1} & \cdots & d_{ij} & d_{in} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & d_{in} & \cdots & d_{ij} & d_{in}
\end{bmatrix}
\]

The measurement of conflict between evidences \( m_i \) and \( m_j \) is:
\[
K(m_i, m_j) = D(d_{i1}, d_{i2}, \cdots, d_{ij-1}, d_{ij+1}, \cdots, d_{in})
\]

A vector can be obtained from the result:
\[
K = (k_{i,1}, k_{i,2}, \cdots, k_{i,j-1}, k_{i,j+1}, \cdots, k_{i,n})
\]

The normalization of evidence conflict is:
\[
k_{ij}^R = \frac{k_{ij}}{\sum_{j(i)} k_{ij}^R}
\]

The evidence entropy of conflict vector \( K_j^R \) after normalization is:
\[
H_i = \sum_{j(i) \neq i} k_{ij}^R \ln(k_{ij}^R) \quad i, j = 1, 2, \cdots n
\]

The weight coefficient of evidence \( E_i \) is as follow:
\[
\omega_i = 1 / H \sum_{j=1}^{n} H_j^{-1}
\]

The weight vector can be fixed from the formula above. If it supposes \( W_{\text{max}} = \{\omega_1, \omega_2, \cdots, \omega_n\} \), the relative weight vector can be shown as follow:
\[
W^* = \{\omega_1, \omega_2, \cdots, \omega_n\} / \omega_{\text{max}}
\]

Therefore, the reliability of the evidence is:
Finally, the adjustment of reliability is as follow:

\[ m_i^* (A_k) = \alpha_i m_i (A_k) \quad k = 1, 2 \cdots r \]

(13)

4 Simulation Result

Experiment one:

- \( e_1 : m_i (A_1) = 0.90, m_i (A_2) = 0, m_i (A_3) = 0.10 \)
- \( e_2 : m_i (A_1) = 0, m_i (A_2) = 0.90, m_i (A_3) = 0.10 \)
- \( e_3 : m_i (A_1) = 0.55, m_i (A_2) = 0.10, m_i (A_3) = 0.35 \)

The fusion result of different method is show as follow:

<table>
<thead>
<tr>
<th>Evidence</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>D-S evidence</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Reference[3]</td>
<td>0.4979</td>
<td>0.3178</td>
<td>0.1843</td>
</tr>
<tr>
<td>The New Method</td>
<td>0.5111</td>
<td>0.3049</td>
<td>0.1840</td>
</tr>
</tbody>
</table>

5 The conclusion

In this paper, it analyzes the shortcoming of DS evidence theory and expounds relative merits of some improved methods. According to them, a new method is presented. It has a lot of advantages such as a simple form, a high precision and reflecting the uncertainty objectively. In addition, it can make a precise judgment when small evidence disturbances exist, which will make the judgment more reliability and validity.

References