Advanced Science and Technology Letters
Vol.31 (MulGraB 2013), pp.143-147
http://dx.doi.org/10.14257/astl.2013.31.32

New Simple Control Algorithms for Modified Function Projective Synchronization

Hongyue Du [1,1], Wei Yao[1], Ge Hu[1], Jian Wang[1]

[1] School of Automation, Harbin University of Science and Technology.
150080 Harbin, PR China
du_hong_yue@163.com

Abstract. This paper investigates some new methods of modified function projective synchronization in chaotic systems. A general simple control law is proposed, which contains only feedback term and easy to implement in practice. Moreover, proposed adaptive method can achieve MFPS even not require any additional information regarding the drive system and the feedback gains of the closed loop control part can be automatically adapted to suitable constants. Numerical example is provided to show the effectiveness of proposed method.

Keywords: Modified function projective synchronization, simple control algorithm, adaptive control, robust control

1 Introduction

Since the seminal work of Pecora and Carroll [1], in which presented a successful method to synchronize two identical chaotic systems, synchronization in chaotic dynamic systems has received a great deal of interest among scientists from various research fields. Up to now, many different synchronization regimes have been studied, which include complete synchronization [1], phase synchronization [2], generalized synchronization [3], lag synchronization [4], time scale synchronization [5], projective synchronization [6], and function projective synchronization [7] etc.

Recently a new type of synchronization, termed as modified function projective synchronization, have been extensively investigated in [8-16], where the drive and response systems could be synchronized to a desired scaling function matrix. The novelty feature of this synchronization phenomenon is that the scaling functions can be arbitrarily designed to different state variables by means of control, while the unpredictability of the scaling functions in MFPS can additionally enhance the security of communications. In Ref. [8], the authors gave the MFPS scheme of two coupled Lorenz systems. Ref. [9] investigated adaptive modified function projective synchronization of hyperchaotic systems with unknown parameters. Based on active control scheme, a general method of MFPS with time delay was investigated in Ref. [10]. Ref. [11] investigated switched modified function projective synchronization of two identical Qi hyperchaotic systems by adaptive control method. Ref. [12]

ISSN: 2287-1233 ASTL

investigates the modified function projective synchronization (MFPS) of drive-response dynamical networks using adaptive open-plus-closed-loop control method. More general forms of MFPS have been extensively investigated in Refs. [13-16].

In most previous proposed references [8-16], the designed controllers contain some nonlinear terms of the systems, which is more complicated and hard to implement in practice. Differ from the ones proposed in [8-16], our designed controller contain only feedback error term, which is simple and easy to implement in practice. Furthermore, proposed adaptive method can achieve MFPS even not require any additional information regarding the drive system and the feedback gains of the closed loop control part can be automatically adapted to suitable constants. To the best of our knowledge, at present, there are few theoretical results about it.

The remainder of this paper is organized as follows: In Section 2, some preliminaries are briefly outlined. The main theorems for MFPS are given in Section 3. In Section 4, we will choose two groups of examples to show the effectiveness of the proposed methods. Conclusions are finally drawn in Section 5.

2 Preliminaries

The drive system and the response system are defined below

$$\dot{\boldsymbol{x}} = \boldsymbol{f}\left(\boldsymbol{x}\right) \tag{1}$$

$$\dot{\boldsymbol{y}} = \boldsymbol{f}\left(\boldsymbol{y}\right) + \boldsymbol{u} \tag{2}$$

where $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{R}$ are the state vectors, $\boldsymbol{f} : \boldsymbol{R}^n \to \boldsymbol{R}^n$ are continuous nonlinear vector functions, $\boldsymbol{u}$ is the vector controller. We define the error vector

$$\boldsymbol{e} = \boldsymbol{\Lambda}\left(t\right)\boldsymbol{x} - \boldsymbol{y} \tag{3}$$

where $\boldsymbol{\Lambda}\left(t\right)$ is a *n*-order diagonal matrix, $\Lambda(t) = diag(\alpha_1(t), \alpha_2(t), \cdots, \alpha_n(t))$ and $\alpha_i(t) \neq 0 \ (i = 1, 2, \cdots, n)$, is a continuously differentiable function with bounded.

Definition 1 (MFPS). For the drive system (1) and the response system (2), it is said that the system (1) and the system (2) are modified function projective synchronization (MFPS), if there exists a scaling function matrix $\boldsymbol{\Lambda}\left(t\right)$ such that $\lim_{t\to\infty}\left\|\boldsymbol{e}(t)\right\| = 0$.

Our goal is to design a simple controller *u* such that the controlled response system (2) could be MFPS to the drive system (1), i.e. $\lim_{t\to\infty}\left\|\boldsymbol{e}(t)\right\| = 0$.

Assumption 1. The norm of $\left\|\dot{\boldsymbol{\Lambda}}\left(t\right)\right\|$ is bounded, that is $\left\|\dot{\boldsymbol{\Lambda}}\left(t\right)\right\| \leq a^*$, where a^* is the upper limit of the norm of $\dot{\boldsymbol{\Lambda}}(t)$.

3 Controller design

In most previous proposed references, the designed controllers contain some nonlinear terms of the systems, which are more complicated and hard to implement in practice. In this section, a general scheme is proposed in Theorem1 and an adaptive feedback error scheme is proposed in Theorem2, which contain only feedback error term and are easy to implement in practice.

3.1 General scheme

Theorem 1. Suppose Assumption 1 holds and $\left\|\mathbf{\Lambda}(t)\boldsymbol{f}(\boldsymbol{x})+\dot{\mathbf{\Lambda}}(t)\boldsymbol{x}\right\|\le M_1$, $\left\|\boldsymbol{f}(\boldsymbol{y})\right\|\le M_2$. For a given synchronization scaling function matrix $\mathbf{\Lambda}(t)$, if $p>M_1+M_2$ then the MFPS between the drive system (1) and the response system (2) will occur by the control law as below

$$\boldsymbol{u}=p\,\mathrm{sgn}(\boldsymbol{e}) \tag{4}$$

where $\boldsymbol{e}=\mathbf{\Lambda}(t)\boldsymbol{x}-\boldsymbol{y}$, k is a arbitrary positive constant and sgn(.) denotes the sign function.

3.2 Adaptive scheme

Although the proposed control law in Theorem1 is simple, the feedback gain is hard to achieve. In this subsection, we will further investigate adaptive feedback gain scheme.

Theorem 2. Suppose Assumption 1 holds. For a given synchronization scaling function matrix $\mathbf{\Lambda}(t)$, the MFPS between the drive system (1) and the response system (2) will occur by the control law as below

$$\boldsymbol{u}=p\,\mathrm{sgn}(\boldsymbol{e}) \tag{5}$$

$$\dot{p}=k\boldsymbol{e}^T\,\mathrm{sgn}(\boldsymbol{e}) \tag{6}$$

where $\boldsymbol{e}=\mathbf{\Lambda}(t)\boldsymbol{x}-\boldsymbol{y}$, k is arbitrary positive constant and sgn(.) denotes the sign function.

4 Illustrative examples

In this section, we choose chaotic Lü and Chen systems as examples to show the effectiveness of the proposed methods.

We take Lü system as the drive system, which is described by

$$\begin{cases}\dot{x}_1=a\left(x_2-x_1\right)\\ \dot{x}_2=-x_1x_3+cx_2\\ \dot{x}_3=x_1x_2-bx_3\end{cases} \tag{7}$$

where x_1, x_2, x_3 are state variables, a, b, c are system parameters. When three real parameters $a = 36, b = 3, c = 20$, the system shows chaotic behavior.

The controlled Chen system, as the response system, is described as

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1) + u_1 \\ \dot{y}_2 = -y_1 y_3 + c y_2 + u_2 \\ \dot{y}_3 = y_1 y_2 - b y_3 + u_3 \end{cases} \tag{8}$$

where y_1, y_2, y_3 are state variables, u_1, u_2, u_3 are the controllers, which can be designed by Theorem 2. When three real parameters $d = 35, e = 3, f = 28$, the system shows chaotic behavior.

References

1. Pecora LM, Carroll TL: Synchronization in chaotic systems. Phys. Rev. Lett. 64, 821--824 (1990)
2. Roseenblum MG, Pikovsky AS, Kurths J: Phase synchronization of chaotic oscillators. Phys. Rev. Lett. 76, 1804--1807 (1996)
3. Rulkov NF, Sushchik MM, Tsimring LS: Generalized synchronization of chaos in directionally coupled chaotic systems. Phys. Rev. E 51, 980--994 (1995)
4. Rosenblum MG, Pikovsky AS, Kurths J: From phase to lag synchronization in coupled chaotic oscillators. Phys. Rev. Lett. 78, 4193- -4196 (1997)
5. Hramov A. E., Koronovskii A. A.: An approach to chaotic synchronization, Chaos 14, 603--610 (2004)
6. Mainieri R., Rehacek J.: Projective synchronization in three-dimensional chaotic systems, Phys. Rev. Lett. 82, 3042--3045 (1999)
7. Chen Y., Li X.: Function projective synchronization between two identical chaotic systems, Int. J. Mod. Phys. C 18, 883--888 (2007)
8. Du H., Zeng Q., Wang, C.: Modified function projective synchronization of chaotic system, Chaos Solitons Fractals 42, 2399--2404 (2009)
9. Zheng K., Dong G.: Adaptive modified function projective synchronization of hyperchaotic systems with unknown parameters, Comm. Nonlinear Sci. Numer. Simul. 15, 3547--3550 (2010)
10. Du H., Zeng Q., Lü N.: A general method for modified function projective lag synchronization in chaotic systems. Phys. Lett. A 374, 1493--1496 (2010)
11. Sebastian Sudheer K., Sabir M.: Switched modified function projective synchronization of hyperchaotic Qi system with uncertain parameters, Comm. Nonlinear Sci. Numer. Simul. 11, 4058-4061 (2010)
12. Du H.: Adaptive open-plus-closed-loop control method of modified function projective synchronization in complex networks, Int. J. Mod. Phys. C 22, 1393--1407 (2011)
13. Yu Y., Li H.: Adaptive generalized function projective synchronization of uncertain chaotic systems, Nonlinear Anal. Real World Appl. 11, 2456--2459 (2010)
14. Sebastian Sudheer K., Sabir M.: Adaptive modified function projective synchronization between hyperchaotic Lorenz system and hyperchaotic Lü system with uncertain parameters. Phys. Lett. A 373, 3743--3748 (2009)
15. Chen Y., Cao L., Sun M.: Robust modified function projective synchronization in network with unknown parameters and mismatch parameters, Int. J. Nonlinear Science 10, 17--23 (2010)

16. Fu G.: Robust adaptive modified function projective synchronization of different hyperchaotic systems subject to external disturbance, Comm. Nonlinear Sci. Numer. Simul. 17, 2602--2608 (2012)