Optimization Study on Interval Number Judgment Matrix Weight Vector Based on Immune Evolution Algorithm

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Abstract. In light of the features of interval number judgment matrix, we transform the solution problem of weights into the optimization one of nonlinear with restriction, design immune evolution algorithm on the base of immune mechanism, and use the convergence ability of immune system to search the optimal access to get interval numbers from matrix weight vector. In the evolution process, search can be dramatically improved by overcoming the problems of earliness and degradation during global search. Simulation results of experiments show the advantages of this algorithm in accuracy, convergence, convergence rate and so on.

Key Words: immune evolution, interval number, matrix weight, nonlinear optimization

1 Introduction

Until now, there are many approaches to identifying weights¹, such as deciding weight by experts, deciding weight by AHP, deciding weight by entropy values. However, deciding weight by experts depends on experts’ knowledge and experience, so this approach is very subjective and random; deciding weight by entropy values is objective and precise by using information in various index and calculating information entropy to fix weight according to importance of various index, but this accuracy is difficult to achieve in the real world. Among these approaches, AHP is widely applied to in many field, because this approach adjusts to qualitative and quantitative analysis and policy making by deconstructing hierarchy of target system and restricting experts’ judgment consistently through mean comparison matrix between structural factors. Experts’ experiments, knowledge, instinct and so on play a great role in deciding weight by AHP, but risk evaluation of information security is associated with a complicated issue such as organization management, technology and social environment. Thus, experts only give a fuzzy extent-an interval number to show judgment-to judge problems and fix evaluation data because experts cannot be clear about problem’s essence and and experiment and instinct. Obviously, interval
number adapts to human judgment more. Whereas, traditional AHP is not good at interval number judgment. That is why it should be improved in order to adjust to decide weight of various indicators in the process of risk evaluation of information security. In this part, we use interval number to construct experts judgment matrix to fit to uncertainty of reality and fuzziness of experts’ experiment, explore a new solution approach to weight vector with interval number judgment matrix by means of optimization calculation theory of immune evolution.

2 Classic ahp to solve judgment matrix weight

This approach is based on classic AHP and combine it into solving judgment matrix weight. Saaty proposed a basic solution approach to judgment matrix weight vector, which includes judgment matrix assignment, eigenvector acquisition and consistently testing one or more circulation.

Under the assumption of a group of factors \( A_1, A_2, \ldots, A_n \), matrix can be shown according to relatively important degree of pairwise comparison:

\[
A = \left( a_{ij} \right)_{n \times n} = \begin{pmatrix}
    a_{11} & a_{12} & \cdots & a_{1n} \\
    a_{21} & a_{22} & \cdots & a_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{n1} & a_{n2} & \cdots & a_{nn}
\end{pmatrix}, \text{in which } i, j \in \{1..n\}
\]

In this matrix, \( a_{ij} \) means importance scale of factor \( A_i \) to factor \( A_j \) compared to evaluation goal, and pairwise comparison is qualitative. To quantize as far as possible in making policy, Saaty introduces 1-9 scale method \(^2\) to fix value or types of relative important relationship and to transform qualitative evaluation into quantitative evaluation. Table 1 lists values of all scales and their implication.

Saaty uses Formulation (4) to examine consistency of judgment matrix.

\[
CR(A) = \frac{\lambda_n - n}{(n-1)RI} \leq 0.1
\]

If Formulation (2) can be satisfied, judgment matrix can accept inconsistency; otherwise, judgment matrix which is built at the beginning is not satisfactory and needs reassigning and rectifying until consistency in this matrix can be found. The solution of weight vector in judgment matrix is based on the process in which Formulation (2) adjust judgment matrix appropriately to satisfy consistency examine. This part makes foundation on it and builds optimization model and algorithm of solving interval number judgment matrix weight vector on the basis of immune evolution mechanism.
3 Optimization model and immune evolution algorithm of solving interval number weight vector

Apparently, solution in Formulation (5) and (6) is a problem of complicated optimization. To acquire weight vector as fast as possible, it is key to construct $A_i, i \in M$, that is to say we need to traverse space of certainty judgment matrix which conform to interval number, and realize fast convergence to lower and upper limit of judgment matrix weight vector which meet restriction. In this paper, we use the global convergence ability of immune system to solve interval number of weight vector by means of immune evolution algorithm. In immune system, antibody can testify antigen and assist immune cells to eliminate antigen. Once antibody tests antigen, antibody can generate clonal expansion and then produce lots of antibody. After that, new antibody will experience high frequent mutation. With the elimination of antigen, antibody library of dynamic expansion will remove some antibody with lower affinity to antigen and then achieve stability of antibody group. At the same time, predominant antibody will be remain by immune memory and be able to induce higher secondary immune response. As far as optimization issue mentioned in this part is concerned, target function is antigen while judgment matrix decided randomly is antibody. Through generating all kinds of immune operators and keeping antibody and immune memory, we can find antigen and then solve weight vector of interval number judgment matrix as fast as possible.

4 Design of algorithm and experimental results

According to Sugihara, interval number judgment matrix weight vector should meet the requirement of Formulation (7) and (8),

$$\sum_i w_i - \max_j (w_j - w_i) \geq 1$$  \hspace{1cm} (3)

$$\sum_i w_i + \max_j (w_j - w_i) \leq 1$$  \hspace{1cm} (4)

It is shown in Formulation (3) and (4) that we should try to expand searching range of components of each weight when to solve interval number judgment matrix weight vector. In this way, it is more suitable to express uncertainty of interval number weight. As Table 5 is shown, the results of this part meet Formulation (3) and (4), which means this algorithm is correct and valid. Comparing them with the results of reference [7], we can find the results of this part better reflect scope that uncertainty of weight distribution can get to.

In this experiment, convergence curve of each component of weight vector can be demonstrated by from Fig1 as followings.
Fig.1 convergence curve of each component of weight vector (a) convergence curve of solving lower limit of $w_1$, (b) convergence curve of solving upper limit of $w_1$, (c) convergence curve of solving lower limit of $w_2$, (d) convergence curve of solving upper limit of $w_2$, (e) convergence curve of solving lower limit of $w_3$, (f) convergence curve of solving upper limit of $w_3$. 
5 Conclusion

We can find that algorithm mentioned in this part has very good convergence. In the first 10 generations after initialization, slope of convergence curve is very big, which reflects algorithm in the first 10 generation has high convergence. However, from Generation 10 to Generation 40, curve has faster convergence rate and gradually keeps stable. Based on statistics in immunological memory, interval number in Generation 40 is 90% of Generation 100. From Generation 60 to Generation 100, curve keeps very steady; what’s more, convergence curve of most testing cases keeps horizontal. Comparing Chart 1,3,5,7 with Chart 2;4;6;8, we know that convergence curves of lower limit is more inclined to keep stable than those of upper limits. That is because interval number is bigger than zero, and change scope of lower limit is restricted.

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