Optimal Design of Nonlinear Fuzzy Model by Means of Independent Fuzzy Scatter Partition

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Abstract. We introduce a optimal design of fuzzy model by means of independent fuzzy scatter partition to construct the nonlinear model. The fuzzy rules of fuzzy model are generated by partitioning the input space in the fuzzy scatter form. The premise parameters of the rules are determined by membership matrix by means of FCM clustering algorithm that has independent fuzzification factors. The consequence part of the rules is represented in the form of polynomial functions. And the optimal process is conducted by PSO to design the optimal model. The proposed model is evaluated using the data widely used in nonlinear process.

Keywords: Fuzzy Scatter Partition, Fuzzy Model, Fuzzy C-Means Clustering Algorithm, Independent Fuzzification Factor, Particle Swarm Optimization.

1 Introduction

Fuzzy sets have been widely investigated and the fuzzy model is a popular computing framework based on the concepts of fuzzy sets, fuzzy if-then rules, and fuzzy reasoning [1]. It has found successful applications in a wide variety of fields. Linguistic modeling [2] and fuzzy relation equation-based approach [3] were proposed as primordial identification methods for fuzzy models. The general class of Sugeno-Takagi models [4] gave rise to more sophisticated rule-based systems. In fuzzy modeling, the structure and parameter identification are usually concerned [5],[6]. The designers find it difficult to develop adequate fuzzy rules and membership functions to reflect the essence of the data. The generation of fuzzy rules has the problem that the number of fuzzy rules exponentially increases.

In this paper, we introduce a fuzzy model based on independent fuzzy scatter partition of input space. Independent fuzzy partition realized with fuzzy c-means (FCM) clustering [7] help determine the fuzzy rules of fuzzy model. The premise part of the rules is realized with the aid of the scatter partition of input space generated by FCM clustering algorithms. The number of the partition of input space equals the number of clusters and the individual partitioned spaces describe the rules. The consequence part of the rules is represented by polynomial functions. We also

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optimize the parameters of the fuzzy model using particle swarm optimization (PSO) algorithm [8]. The proposed model is evaluated with numerical experimentation to deal with the nonlinear process.

2 Independent Fuzzy Scatter Partition-based Fuzzy Model

2.1 Premise Identification

The premise part of the FIS is developed by means of the fuzzy c-means clustering algorithm [7]. This algorithm divides the input space by the clusters and each partitioned local space represents the fuzzy rules. Therefore, the number of clusters is equal to the number of rules. This algorithm is aimed at the formation of 'c' clusters (relations) in $\mathbb{R}^n$.

Consider the set $X$, which consists of $N$ data points treated as vectors located in some $n$-dimensional Euclidean space, that is $X=\{x_1,x_2,...,x_N\}$, $x_p \in \mathbb{R}^n$. In clustering we assign patterns $x_p \in X$ into $c$ clusters, which are represented by its prototypes $v_i \in \mathbb{R}^n$. The assignment to individual clusters is expressed in terms of the partition matrix $U = [u_{ip}]$ where

$$\sum_{i=1}^{c} u_{ip} = 1, \quad 1 \leq p \leq N \quad (1)$$

$$0 < \sum_{p=1}^{N} u_{ip} < N, \quad 1 \leq i \leq c \quad (2)$$

The objective function $Q$ guiding the clustering is expressed as a sum of the distances of individual data from the prototypes $v_1, v_2, ..., v_c$,

$$Q = \sum_{i=1}^{c} \sum_{p=1}^{N} u_{ip}^m \|x_p - v_i\|^2 \quad (3)$$

Here $\|\|$ denotes the Euclidean distance; 'm' stands for a fuzzification coefficient, $m \geq 1.0$. The resulting partition matrix is denoted by $U = [u_{ip}]$.

The minimization of $Q$ is realized through successive iterations by adjusting both the prototypes and entries of the partition matrix, that is $min Q(U,v_1,v_2, ..., v_c)$. The corresponding formulas used in an iterative fashion read as follows.

$$v_i = \sum_{p=1}^{N} u_{ip}^m x_p / \sum_{p=1}^{N} u_{ip}^m \quad (4)$$

$$u_{ip} = \left( \sum_{j=1}^{c} \left( \frac{\|x_p - v_j\|}{\|x_p - v_i\|} \right)^{\frac{1}{m-1}} \right)^{-1} \quad (5)$$
The resulting partition matrix obtained from (5) becomes the firing strengths of fuzzy rules.

The identification of the conclusion parts of the rules deals with a selection of their structure that is followed by the determination of the respective parameters of the local functions occurring there. The conclusion is expressed as follows.

\[ R^j : \text{If } x_1, \ldots, x_j \text{ is } F_j \text{ Then } y_j = f(x_1, \ldots, x_j). \]  

(6)

Type 1 (Simplified Inference): \[ f = a_{j0} \]

Type 2 (Linear Inference): \[ f = a_{j0} + \sum_{k=1}^{d} a_{jk} x_k \]

Where \( R^j \) is the \( j \)-th rule, \( x_k \) represents the input variables, \( F_j \) is a membership grades (matrix) obtained by using FCM clustering algorithm, \( a \)'s are coefficient of polynomial function.

The calculations of the numeric output of the model, based on the activation (matching) levels of the rules there, are carried out in the well-known format

\[ y^* = \frac{\sum_{j=1}^{c} w_j y_j}{\sum_{j=1}^{c} w_j}, \quad \hat{y} = \frac{\sum_{j=1}^{c} \hat{w}_j y_j}{\sum_{j=1}^{c} \hat{w}_j} \]  

(7)

3 Optimization of the Proposed Model

Particle swarm optimization (PSO) [8] was proposed by Kennedy, Eberhart to simulate social behavior by representing the movement of a bird flock or fish school. PSO is a computational algorithm that optimizes a given problem by iteratively trying to improve candidate solutions (particles). PSO algorithm optimizes a given problem by having a swarm of particles and moving these particles around in the search space. Each particle's movement is affected by its local best positions and is also guided toward the global best positions in the search-space over the particle's position and velocity. And these positions are updated as better positions. The swarm move toward the best solutions. In order to optimize the parameters of the proposed model, we determined the fuzzification factors of the clusters composed of the premise part of the fuzzy rules.

4 Experimental Studies

We discuss numerical example in order to evaluate the advantages and the effectiveness of the proposed approach. The time series data (296 input-output pairs) coming from the gas furnace nonlinear process has been intensively studied in the previous literature [9]. The delayed terms of methane gas flow rate \( u(t) \) and carbon dioxide density \( y(t) \) are used as input variables organized in a vector format as \( [u(t-3), \ldots] \),...
$u(t-2), u(t-1), y(t-3), y(t-2), y(t-1)$. $y(t)$ is the output variable. The first part of the data set (consisting of 148 pairs) was used for training purposes. The remaining part of the series serves as a testing data set. We consider the MSE as a performance index.

We construct the model for a two-dimensional system by configuring 2-input 1-output system using $u(t-3)$ and $y(t-1)$ as inputs. And we experimented with the model using the parameters outlined in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generation</td>
<td>150</td>
</tr>
<tr>
<td>Swarm size</td>
<td>50</td>
</tr>
<tr>
<td>$V_{\text{max}}$</td>
<td>20%</td>
</tr>
<tr>
<td>Inertia weight</td>
<td>[0.4, 0.9]</td>
</tr>
<tr>
<td>Acceleration constants</td>
<td>2.0</td>
</tr>
</tbody>
</table>

We experimented by optimizing the parameters of the fuzzification factors of the clusters in two methods using PSO. One tuning method is a method that all clusters have the same fuzzification factor and another tuning method is a method that each cluster has the independent fuzzification factor.

Table 2 summarizes the performance index for training and testing data by setting the number of clusters and inference type. Here, PI and E_PI stand for the performance index for the training data set and the testing data set, respectively.

This table shows that the proposed approach has better results. From Table 2, we selected the best model with five rules (clusters) with linear inference that exhibits $\text{PI} = 0.019$ and $\text{E}_\text{PI} = 0.285$.

<table>
<thead>
<tr>
<th>No. of Clusters</th>
<th>(a) Simplified Inference</th>
<th>(b) Linear Inference</th>
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<tbody>
<tr>
<td></td>
<td>Same fuzzification factor</td>
<td>Independent fuzzification factors</td>
</tr>
<tr>
<td></td>
<td>PI</td>
<td>E_PI</td>
</tr>
<tr>
<td>2</td>
<td>2.225</td>
<td>2.431</td>
</tr>
<tr>
<td>3</td>
<td>0.946</td>
<td>1.376</td>
</tr>
<tr>
<td>4</td>
<td>0.564</td>
<td>1.280</td>
</tr>
<tr>
<td>5</td>
<td>0.592</td>
<td>1.161</td>
</tr>
</tbody>
</table>
Figure 1 shows independent fuzzy-partitioned input spaces using FCM clustering algorithm for the selected model. Figure 2(a) depict that the selected model has three clusters because the other two clusters in the selected model didn’t have the patterns.

Figure 2 presents the optimization procedure for five particles and performance index for the selected model using PSO. The model outputs of training data and testing data for the selected model are presented in figure 3.

Figure 1. Independent partitioned input spaces using FCM clustering algorithm.

Figure 2. Optimization process.

Figure 3. Original and model outputs.
5 Conclusions

In this paper, we introduced a fuzzy model based on independent fuzzy scatter partition of input space. The input spaces of the proposed model were divided as the scatter form using FCM clustering algorithm to generate the rules of the system for nonlinear process. And the proposed fuzzy model was optimized by PSO to find the best values of the fuzzification factors to reflect the characteristics of the data. By this method, we could alleviate the problem of the curse of dimensionality and design the fuzzy model that is compact and simple. From the results in the previous section, we were able to design preferred model with a very small number of rules that has the approximation abilities and the generalization capabilities.

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References