Sliding Mode Control for a Plant with a Time Delay

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Abstract. In this study, we propose an advanced sliding mode control (SMC) technique for plants with time delays. In control engineering, the presence of time delays in a control system may cause it to become unstable. Furthermore, SMC cannot control systems having time delays. To solve this problem, we introduce a simple plant predictor to the SMC method. Therefore, our proposed method is designed easily. Our simulation result shows that the proposed method is effective for plants with time delays. In our experiment, we confirm the practicality of the proposed method by applying a pneumatic actuator.

Keywords: sliding mode control, time delay, plant predictor, pneumatic actuator

1 Introduction

In control engineering, it is often problematic to maintain that control because of errors in the transfer function of the equipment due to either deterioration or changes in the environment of the equipment used. For several years, researchers have attempted to solve this problem by studying robust control, and various control methods have been proposed. Sliding mode control (SMC) is one such robust control method that is based on the concept of a variable structure control system.

SMC has a switching surface, which is called a switching hyperplane, and it is used to stabilize a system [1] - [3]. In addition, the sliding mode controller can ensure good robustness by having both linearity and nonlinearity. However, in SMC, there is high-frequency vibration known as chattering. This phenomenon is noticeable in systems having time delays [4]. As the time delay increases, the ability to maintain control becomes more difficult. Because there are often time delays in real systems, it is important to solve this problem in practical applications of SMC.

In this study, we control a system having a time delay by adding a predictor to SMC. In the simulation study, we confirm the effectiveness of our proposed method.
We also conducted actual experiments using a pneumatic actuator, and showed the practicality of the proposed method.

2 Controlled System

In this paper, the state space equation of a controlled plant, which is controllable and observable, is expressed by equation (1). This plant includes a time delay.

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t - \tau) \\
y(t) &= c\nu(x(t))
\end{align*}
\tag{1}
\]

where \(y(t)\) is the plant output, \(u(t)\) is the plant input, \(\tau\) is the time delay, and \(x(0) = 0\).

3 Servo System

The state space equations are designed on the basis of a spreading system as follows.

\[
\begin{align*}
\dot{x}_s(t) &= A_s x_s(t) + b_s u(t - \tau) \\
y_s(t) &= c_s x_s(t)
\end{align*}
\tag{2}
\]

\[
A_s = \begin{bmatrix} A & 0 \\ -c & 0 \end{bmatrix}, \quad b_s = \begin{bmatrix} b \\ 0 \end{bmatrix}, \quad c_s = [0 \ 1], \text{ and } x_s(t) = \begin{bmatrix} x(t) \\ z(t) \end{bmatrix}.
\]

Let

\[
z(t) = \int (r(t) - y(t)) \, dt, \quad \dot{z}(t) = r(t) - y(t).
\]

where \(r(t)\) is a reference signal.

The feedback gain \(F_1\) and servo gain \(F_2\) are introduced by minimizing the cost function in equation (3). These gains are given by the optimal control method required to minimize the cost function.

\[
J = \int_0^\infty \{x_s(t)^T Q_s x_s(t) + r_s u(t)^T\} dt,
\tag{3}
\]

where \(Q_s \in \mathbb{R}^{(n+1) \times (n+1)}\), \(r_s \geq 0\). The feedback gain \(F_1\) and servo gain \(F_2\) are rewritten as \(F = [F_1 \ F_2]\). \(F\) is given by equation (4).

\[
F = -r_s^{-1} B_s^{-1} P_s,
\tag{4}
\]

The matrix \(P_s\) is the solution of the Riccati equation and is given by equation (5).

\[
P_s A_s + A_s^T P_s - P_s B_s^T r_s^{-1} B_s P_s + Q_s = 0, \quad P_s \in \mathbb{R}^{n+1 \times (n+1)}.
\tag{5}
\]
4 Sliding Mode Control

SMC [5][6] is designed by broadly the switching hyperplane and sliding mode controller. The SMC system is expressed as follows.

\[
\begin{aligned}
    x(t) &= A x(t) + b u(t) \\
    \sigma(t) &= S x(t)
\end{aligned}
\]

where \(\sigma(t)\) is a switching function and \(S\) is the slant of the switching hyperplane.

4.1 Switching Hyperplane

In this section, we explain the design of the switching hyperplane. The slant \(S\) is required when designing a switching hyperplane, and is determined by solving the Riccati equation employing a method that uses the system’s zero-point. The Riccati equation is given by equation (7).

\[
P A + A^T P - P B R^{-1} B^T P + Q = 0, S = B^T P,
\]

where \(A = A + \varepsilon I \in \mathbb{R}^{n \times n}\), \(P \in \mathbb{R}^{n \times n}\), and \(r_3 \in \mathbb{R}^{\varepsilon \times \varepsilon}\)

On the basis of \(S(sI - A^T + BS)^{-1}B\) becomes strictly positive real; thus, its zero-point is stable.

4.2 Sliding Mode Controller

The control input \(u(t)\) in the sliding mode controller consists of a linear input \(u_l(t)\) and a nonlinear input \(u_{nl}(t)\). The control input \(u(t)\) is expressed as follows.

\[
u(t) = u_l(t) + u_{nl}(t) = -(SB)^{-1} S A x(t) - k \frac{\sigma(t)}{[\sigma(t)] + \delta}, \delta \in \mathbb{R}^{\varepsilon \times 1}, \delta > 0
\]

where \(u_l(t)\) is a control system of an equal value \(k\) is the nonlinear input gain.

4.3 Prevention of the chattering phenomenon

In fact, if the sliding mode controller is designed using \(u_{nl}(t)\) in equation (8), chattering occurs. When \(\sigma(t) = 0\) for a nonlinear input, \([\sigma(t)]\) in the denominator becomes 0, and quick switching of the input occurs. To avoid chattering, we rewrite equation (8) as (9).

\[
u(t) = u_l(t) + u_{nl}(t) = -(SB)^{-1} S A x(t) - k \frac{\sigma(t)}{[\sigma(t)] + \delta}, \delta \in \mathbb{R}^{\varepsilon \times 1}, \delta > 0
\]
In this section, we explain the design method for a plant predictor in continuous-time [7]. The space state equations in discrete-time are obtained by discretizing equation (1) using bilinear transform. We calculate the plant predictor for these equations.

\[
\begin{align*}
x(k + 1) &= Ax(k) + bu(k - d) \\
y(k) &= cx(k)
\end{align*}
\]

where \(d\) represents the time delay. In equation (10), we apply the substitution \(k = k + 1\), and obtain the state of the plant at a sampling time of \(k + 2\). We obtain the state of the plant at a sampling time \(k + d\) by repeating this calculation.

\[
\tilde{x}(k + d) = A^d x(k) + \sum_{i=1}^{d} A^{d-i} bu(k - d + i - 1)
\]

The predicted output of the plant is

\[
\hat{y}(k + d) = cx(k + d).
\]

Finally, we convert equation (11) in continuous time. Because these predicted state variables are not observed directly, an observer estimates the state variables of the plant. \(L_o\) in equation (13) represents the observer gain.

\[
\begin{align*}
\dot{\hat{x}}(t) &= A \dot{\hat{x}}(t) + bu(t - \tau) + L_o (y(t) - \hat{y}(t)) \\
\dot{\hat{y}}(t) &= c \hat{x}(t)
\end{align*}
\]

6 Proposed Method

Fig. 1 shows a block diagram of the proposed method. The proposed method consists of SMC and the plant predictor.
7 Simulation Results

In this section, the effectiveness of the proposed method is shown by simulation studies. In this simulation, we assume the following plant parameters with a time delay. The sampling time $T_s$ is 10 [ms], and a step input of 5.0/s is introduced at $t = 0$ [s]. We introduce a +50% error in the plant parameters $A$ and $b$ to confirm the robustness of the proposed method.

$$G(s) = \frac{1}{0.229s^{0.2111}}$$

(14)

The parameters of the proposed method in this simulation are as follows: $Q_{se} = 0.01$, $r_{se} = 1$, $F = [0.214 \ 0.100]$, $Q_s = 100$, $r_s = 1$, $S = 10.231$, $\varepsilon = 1$, $k = 5$, $\delta = 100$, and $L_o = 436.681$. Fig. 2(a) shows the simulation result of the proposed method.

8 Experiment Result

In this section, we experimentally confirm the practicality of the proposed method using a pneumatic actuator. The transfer function of the pneumatic actuator is given in equation (14). The actuator varies from 0° - 90°, and the angle is detected by a sensor and outputs from −10 to 10 [V], depending on the angle. The air pressure is 0.4 [Mpa].

In this experiment, the sampling time $T_s$ is 10 [ms], and the step input 5.0/s is introduced at $t = 0$ [s]. In addition, the parameters of the proposed method are as follows: $Q_{se} = 0.05$, $r_{se} = 1$, $F = [0.320 \ 0.223]$, $Q_s = 100$, $r_s = 1$, $S = 10.231$, $\varepsilon = 1$, $k = 7$, $\delta = 100$, and $L_o = 436.681$. Fig. 2(b) shows the experiment result.
9 Conclusion

In this study, we constructed a robust control system corresponding to a system with a time delay by introducing a predictor to control the SMC method. The proposed method is shown to control many process systems such as chemical and thermal process. Moreover, the system is simple to construct. In the simulation study, we have shown the effectiveness of the proposed method. Furthermore, we confirmed the practicality of the proposed method by adapting it to a pneumatic actuator.

References