Performance Analysis for DF relay networks with PSA-CH schemes

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Abstract. This paper provides a framework for evaluating the error rate performance of decode and forward (DF) relay cooperative transmission in the presence of channel estimation error (CHE) caused by pilot symbol assisted-channel estimation (PSA-CE) schemes. Simulation results are finally presented to validate derived ABER and ASER expressions.

Keywords: DF relay, CHE, PSA-CE, MGF, ABER, ASER.

1 Introduction

Cooperative diversity has recently been widely discussed for wireless networks [1] [2]. In [5] the authors provided a framework for evaluating the BER performance of AF relay-assisted cooperative transmission in the presence of imperfect channel estimation. Nevertheless, a framework in [5] does not include PSA-CE schemes which can be applied in practical systems so that error rate performance gives error-floor even at high SNRs.

So far as we know, the general approach based on PSA-CE schemes for DF relaying has not been addressed in the literature yet. Furthermore, no one has expressed average error rate expressions with PSA-CHE, which can be explain how an erroneous detection at each relay and the channel estimation errors affect both the received signal-to-noise ratio (SNR) and the averaged BER. The remainder of the paper is organized as follows: Section II describes the system model for AF relaying.

2 DF Relay Networks

Let us consider DF relay systems with a source(S), a destination(D), and a relay(R). The number of relays is R. It is assumed that S and R transmit over orthogonal channel. For the rth relay, let $h_{DS}$, $h_{RS}$, and $h_{DR}^r$ be the channel gains of S-D, S-R,
and R-D link channels, respectively. In this paper, wireless channels between any pair of nodes are assumed quasi-static INID Rayleigh fading. The received signal for S-R links can be presented as $y_{RS}^r[t] = h_{RS}^r s^r[t] + n_{RS}^r[t]$. For PSA-CE schemes, the channel coefficients can be estimated over quasi-static fading channels as

$$h_{RS}^r = \frac{1}{N_p^r} \sum_{r=1}^{N_p^r} s^r[t] y_{RS}^r[t] = h_{RS}^r + e_{RS}^r$$

where $e_{RS}^r = \frac{1}{N_p^r} \sum_{r=1}^{N_p^r} s^r[t] n_{RS}^r[t]$ is the channel estimation errors (CHE). Then, the decision variable can be written as $z_{RS}^r[t] = \left(h_{RS}^r\right)^* y_{RS}^r[t]$ and the received instantaneous SNR is expressed as $\gamma_{R+r} = \frac{|h_{RS}^r|^2}{\sigma^2 (N_p^r + 1) / N_p^r}$. In addition, the average SNR are defined as $\overline{\gamma}_{R+r} = E[\gamma_{R+r}] = \frac{E[|h_{RS}^r|^2]}{\sigma^2 (N_p^r + 1) / N_p^r}$. Note that For S-D and R-D links, same derivation can be applied. The averaged SER of the $r$ th S-R link can be written for MPSK as

$$P_{S}\left(\overline{\gamma}_{R+r}\right) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} M_{\overline{\gamma}_{R+r}}(s) \, d\theta$$

with $M_{\overline{\gamma}_{R+r}}(s) = \left(1 + s \overline{\gamma}_{R+r}\right)^{-1}$ [2] [3].

3 Performance analysis of DF Relay Systems with PSA-CHE

In DF relay systems, the $r$ th relay participates in transmitting the regenerated symbol of $\hat{s}_r[t]$ only when messages are correctly decoded.

3.1 Error-Event of Relay Nodes

In order to generally derive the analytical method based on error-events at relays, let us define the $p$ th error-event vector $E^p$ as $E^p = \left[e_1^p \cdots e_r^p \cdots e_R^p \right]$ [1] with $p \in \{1, 2, \cdots, 2^R\}$ and the total number of error-events is $2^R$. Generally, we can define that $E^1$ is all-zero vector, $E^{2^R}$ is all-one vector, and so on. Note that for the $p$ th error-event, $e_p = 0$ means the correct detection at the $r$ th relay and
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\[ \hat{s}_r[t] = s[t] \] for \( t \in \{N_p + 1, N_p + 2, \cdots, N_F \} \) with the probability of \( 1 - P_S(\bar{r}_r) \). Also, \( e_r^p = 1 \) leads to \( \hat{s}_r[t] = 0 \) with the probability of \( P_S(\bar{r}_r) \). Furthermore, the probability of the \( p \) th error-event at DF relay systems is presented as

\[ P_r^p = \prod_{r=1}^{R} \left[ 1 - P_S(\bar{r}_r) \right]^{\gamma_r^p} \left[ P_S(\bar{r}_r) \right]^{1 - \gamma_r^p} \]  

(3)

with \( \gamma_r^p = (e_r^p + 1) \bmod 2 \) [1].

### 3.2 Average SER Expression

By utilizing the moment generating function (MGF), the average SER can be derived. Conditional symbol error rate for M-PSK are as follows:

\[ P_{S,DF}^p(\gamma_{DF}^p) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp \left( \frac{g_{PSK} \gamma_{DF}^p}{\sin^2 \theta} \right) d\theta \]  

(4)

with \( g_{PSK} = \sin^2(\pi / M) \) and the average symbol error rate can be presented as

\[ P_{S,DF}^p \left( \frac{1}{\pi} \int_0^{(M-1)\pi/M} \prod_{r=0}^{R} M_{\gamma_r^p}^{-1} (s) d\theta \right) \]  

(5)

with \( s = g_{PSK} / \sin^2 \theta \) and \( M_{\gamma_r^p}^{-1} (s) = \left( 1 + s \sum_{e_r^p} \gamma_r^p \right)^{-1} \). Then, taking into account for all the possible error-events, the averaged BER is presented as

\[ P_{S,DF} = \sum_{p=1}^{R} \Pr_r^p \cdot P_{S,DF}^p. \]  

(6)

![Fig. 1. Averaged BER versus SNR for DF relay systems with respect to different number of pilot symbols (M=2, R=1, N_p=0,2,4,8).](image-url)
4 Numerical and Simulation Results

Fig. 1 shows the averaged BERs versus SNR for the DF relay systems. Note that $N_P=0$ means the ideal channel estimation in which analytical results exactly match with simulated ones. From this figure, we can find that there are mismatches between analytical results and simulated ones. Those mismatches can be caused by the Gaussian approximation for CHE which is related with the derivation of received SNR. Furthermore, those mismatches decrease in proportion to the increase of $N_P$. Consequently, it is confirmed that the derived analytical approach can be used as a general tool to verify effects of PSA-CHE on the averaged error rate and cooperative diversity gain over quasi-static Rayleigh fading channels.

5 Conclusions

In this paper, we have developed an analytical method for average error rate expression related with PSA-CHE based on error-events at relay nodes of DF relay systems over quasi-static Rayleigh fading channels. The well-known form error rate expressions were evaluated as both average BER and SER. Moreover, the feed-forward PSA-CHE for S-R link was shown to be a significant factor for practical environments. For the case of a large number of pilots, we were able to verify that our approach well matches with the simulation results. Under the general cooperative diversity scheme, we expect our theoretical results to be useful for further analysis and in the evaluation of practical implementations.

References