Discrete Rule-Based Local Search Metaheuristic for Bézier Curve Parameterization

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Abstract. This paper introduces a new method for solving the parameterization problem of the Bézier curve of a certain degree that approximates a given collection of data points. Our method is based on a discrete based-on-rules local search metaheuristic approach for optimization problems. The method is applied to some simple yet illustrative examples for the cases of 2D and 3D curves. Our experimental results show that the presented method performs very well, being able to fit the data points with a high degree of accuracy.

1 Introduction

Fitting curves to data points is a very important issue in many applied fields such as computer-aided design & manufacturing (CAD/CAM), virtual reality, medical imaging, computer animation, and many others [2, 4, 6, 13, 15, 18]. In all those cases, it is desirable to obtain the fitting curve that approximates the set of data points optimally in the sense of least-squares.

In real-world settings, polynomial curves are usually applied to tackle this issue. Best approximation methods make commonly use of least-squares techniques [5, 17]. In this paper we focus particularly in the case of Bézier curves, where the goal is to obtain the control points of the optimal fitting curve. This problem is far from being trivial, because the curve is parametric. This means that we are confronted with the problem of obtaining a suitable parameterization of the data points [1, 7], thus leading to a difficult over-determined nonlinear problem.

Some recent papers have shown that the application of Artificial Intelligence (AI) techniques can achieve remarkable results regarding this parameterization problem [2–7, 9, 12–16, 19]. In this context, the present paper introduces a new method for solving the parameterization problem of the Bézier curve of a certain degree that approximates a given collection of data points. Our method is based on a discrete local search metaheuristic approach for optimization problems called tabu search, described briefly in Sect. 2. The method is described in Sect. 3 and then applied in Sect. 4 to some simple yet illustrative examples for the cases of 2D and 3D curves. Our experimental results show that the presented method performs very well, being able to fit the data points with a high degree of accuracy. The paper closes with the main conclusions of this contribution and our plans for future work in the field.

2 Tabu Search

Tabu Search is a metaheuristic technique originally developed by Fred W. Glover in 1986 to allow local search methods to overcome local optima [10]. The basic principle of tabu search is to pursue discrete local search whenever a local optimum is reached by allowing non-improving moves. In order to prevent the algorithm to reach into previously visited solutions (thus falling into a cycle), the method make use of memory structures, called *tabu lists*, whose aim is to record the recent history of the search. Basically, a tabu list is a set of banned solutions used to filter the solutions that will be admitted.

The search process can also be enriched with additional rules to drive the search towards promising areas of the search space as well as to promote diversity, playing the role of intermediate and long-term memories, respectively. In order to improve the effectiveness of tabu search, the tabu lists can contain attributes rather than (or, in addition to) solutions. However, this variant might introduce a new problem: tabu lists based on attributes may prohibit attractive moves, even when there is no danger of cycling, or they may lead to an overall stagnation of the searching process. Therefore, we need a procedure to revoke (cancel) tabus by allowing a move, even if it is tabu, to prevent these situations to happen. This is called the *aspiration criteria*.

Once all tabu rules, lists and attributes are determined, the tabu search algorithm is executed until a termination criterion is satisfied. The most commonly used stopping criteria in tabu search are: (1) after a fixed number of iterations (or a fixed amount of CPU time); (2) after some number of iterations without any improvement in the objective function value (the criterion used in most implementations); or, (3) when the objective reaches a pre-specified threshold value, which is problem-dependent and usually specified by the user. The reader is referred to [11] for more details about the tabu search method, its variants, advantages and limitations.

3 Our method

We assume that the reader is familiar with the main concepts of free-form parametric curves [1]. A *free-form parametric Bézier curve* $\mathbf{C}(t)$ *of degree* n is defined as:

$$\mathbf{C}(t) = \sum_{j=0}^{n} \mathbf{P}_j B_j^n(t) \quad (1)$$

where $\mathbf{P}_j$ are vector coefficients (usually referred to as the *control points*), $B_j^n(t)$ are the *Bernstein polynomials of index j and degree n*, and t is the *curve parameter*, defined on a finite interval $[0, 1]$. Note that in this paper vectors are denoted in bold. By convention, $0! = 1$.

Let us suppose now that we are given a set of data points $\{\mathbf{Q}_i\}_{i=1,\dots,m}$ in $\mathbb{R}^d$ (usually $d = 2$ or $d = 3$). Our goal is to compute the control points $\mathbf{P}_j$, $(j = 0, ..., n)$ of the approximating curve $\mathbf{C}(t)$ by minimizing the least-squares error, E, defined as the sum of squares of the residuals:

$$E = \sum_{i=1}^{m} \left(\mathbf{Q}_i - \sum_{j=0}^{n} \mathbf{P}_j B_j^n(t_i) \right)^2 . \qquad (2)$$

with $\mathbf{Q}_i = \mathbf{C}(t_i)$, $i = 1, \dots, m$. Considering vectors $\mathbf{B}_j = (B_j^n(t_1), \dots, B_j^n(t_m))^T$, $j = 0, \dots, n$, where $(.)^T$ means transposition, and $\bar{\mathbf{Q}} = (\mathbf{Q}_1, \dots, \mathbf{Q}_m)$, Eq. (2) becomes the following system of equations:

$$\begin{pmatrix} \mathbf{B}_0^T.\mathbf{B}_0 & \dots & \mathbf{B}_n^T.\mathbf{B}_0 \\ \vdots & \vdots & \vdots \\ \mathbf{B}_0^T.\mathbf{B}_n & \dots & \mathbf{B}_n^T.\mathbf{B}_n \end{pmatrix} \begin{pmatrix} \mathbf{P}_0 \\ \vdots \\ \mathbf{P}_n \end{pmatrix} = \begin{pmatrix} \bar{\mathbf{Q}}.\mathbf{B}_0 \\ \vdots \\ \bar{\mathbf{Q}}.\mathbf{B}_n \end{pmatrix} . \qquad (3)$$

This is a strongly nonlinear problem, with a high number of unknowns for large sets of data points, a case that happens very often in practice. Solving the parameterization problem is the key to obtain an optimal fitting Bézier curve of data. In this paper we consider a tabu list L comprised of a fixed number N of banned solutions, N being a parameter of the method. The potential solutions are a collection of parametric vectors for the given data points. These parametric vectors are sorted according to the functional E and the best d solutions are selected for the tabu list L. Then, the best current solution undergoes further transformations, where a random integer index is chosen; then, the component of such index is perturbed additively with a uniform random variable in a given interval. This operation is repeated p times, where p is also a parameter of the method. The resulting vectors are then inserted into the list of potential solutions, which are subsequently ranked according to E, and the tabu list is updated accordingly. The process is performed iteratively for a given number of iterations, until the convergence of the minimization of the error is achieved.

4 Experimental results

This section discusses the performance of our tabu search-based method for Bézier curve parameterization through two simple yet illustrative examples for the cases of 2D and 3D data points. Many other examples have also been tested with excellent results in all cases. They are not reported here because of limitations of space.

Figure 1 shows an example of two-dimensional curve called the *epicycloid*. As the reader can see, the curve has challenging features: several non-differentiable

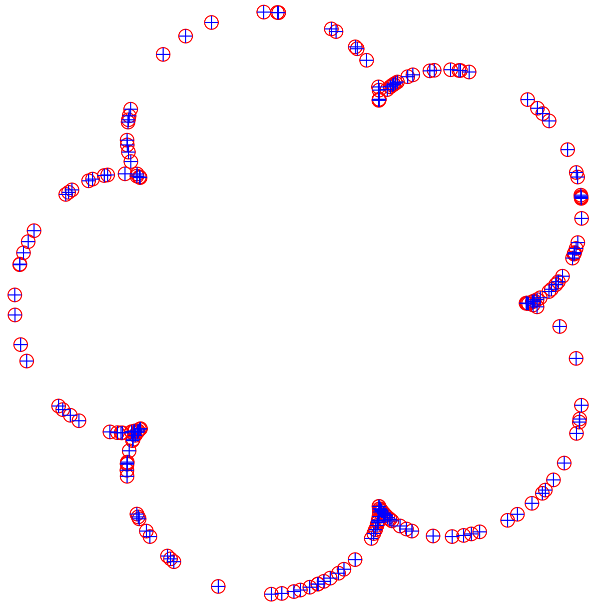

Fig. 1. Applying the tabu search algorithm to a 2D Bézier curve parameterization: original (red emptied circles) and reconstructed (blue + symbols) data points for the epicycloid curve.

turning points. In our figures, the original data points are displayed as red emptied circles whereas the reconstructed points appear as blue plus symbols. The number of data points is $m = 200$. An initial population of 100 randomly chosen solutions is considered; then, the method is executed for 50 iterations. Note the excellent matching between the original and the reconstructed data points.

Figure 2 shows an example of three-dimensional curve: the *Viviani curve*. We applied our method to this example for the same parameter values with $m = 100$ data points. Once again, we obtained an excellent matching between the original and the fitted data points, which is clearly visible in the figure.

All computations in this paper have been performed on a 2.9 GHz. Intel Core i7 processor with 8 GB. of RAM. The source code has been implemented by the authors in the native programming language of the popular scientific program *Matlab*, version 2010b.

5 Conclusions and future work

This paper presents a new method for solving the parameterization problem of the Bézier curve of a certain degree that approximates a given collection of data points. Our method is based on a metaheuristic approach for optimization

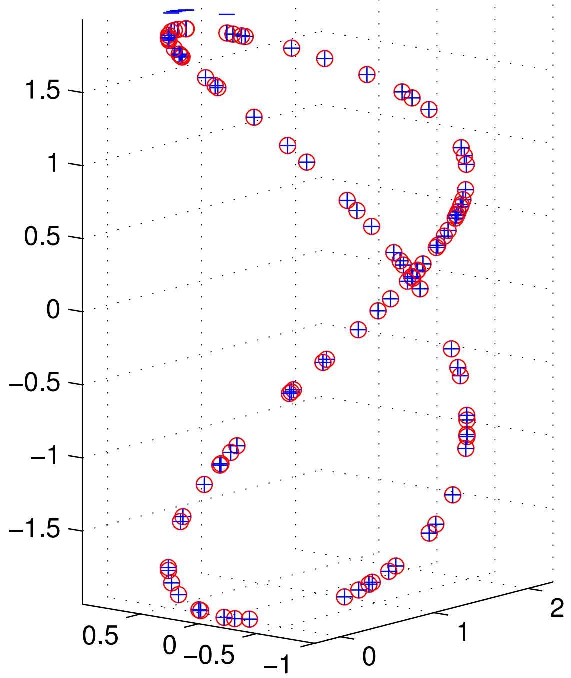

Fig. 2. Applying the tabu search algorithm to a 3D Bézier curve parameterization: original (red emptied circles) and reconstructed (blue + symbols) data points for the Viviani curve.

problems called tabu search. This problem is far from being trivial as soon as no parameterization of data points is assumed *a priori*. Furthermore, data points are subjected to noise in their parametric values (this fact is clearly noticeable from the simple visual inspection of the uneven distribution of data in Figs. 1 and 2), meaning that the uniform parameterization is by no means a feasible solution; instead, a proper parameterization is actually required. The method is applied to some simple yet illustrative examples for the cases of 2D and 3D curves. Our experimental results show that the presented method performs very well, being able to fit the data points with a high degree of accuracy.

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