Switching Control for Uncertain Time-varying Nonlinear Systems using Multi-diffeomorphism

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Abstract. A nonlinear system can be transformed into an equivalent linear system when it satisfies certain conditions. The freedom in the feedback linearization technique is used to improve the transient behavior of the nonlinear system. So, by the switching control with the proper switching rule, we can obtain the better transient behavior. In this paper, we propose a switching rule for taking the transient behavior improvement for the uncertain time-varying nonlinear system. The proposed switching controller can compensate the effect of the uncertainties of the nonlinear system.

Keywords: Multi-diffeomorphism, Feedback Linearization, Uncertainty, Switching Control

1 Introduction

Feedback linearization technique is one of the most effective methods in the field of nonlinear control. In order to use feedback linearization technique, we should solve a series of PDEs and get the solution named diffeomorphism. Feedback linearizable system has various diffeomorphisms and this freedom in linearization method makes it possible to improve the transient behavior of the nonlinear system [1]. By using this property, [2] proposes the switching strategy to get better transient response.

In this paper, by extending the result of [2], we propose a switching rule for the uncertain time-varying nonlinear system. First, we design the feedback controllers to compensate the uncertainties using Lyapunov redesign. After design the controllers, the switching rule for the nonlinear system is proposed.

2 Problem Formulation

Consider the following single-input uncertain time-varying system

$$\dot{x} = f(x,\theta(t)) + \Delta f(x,\theta(t)) + (g(x,\theta(t)) + \Delta g(x,\theta(t)))u \tag{1}$$

where $\theta(t) \in D_\theta \subset R^P$ is a vector of time-varying parameters, $f(x,\theta(t))$, $\Delta f(x,\theta(t))$,

$g(x,\theta(t))$, $\Delta g(x,\theta(t))$ are smooth on R^n. $\Delta f(x,\theta(t))$, $\Delta g(x,\theta(t))$ are uncertainties or unmodeled parts of the system.

Assumption 1. The following system is input-state feedback LTIsable [3].

$$\dot{x} = f(x,\theta(t)) + g(x,\theta(t))u \tag{2}$$

Let $z = \hat{T}(x)$ is a time-varying diffeomorphism such that it transforms (2) into a linear system. Then, by $z = \hat{T}(x)$ and a feedback controller $u = \alpha(x,\theta(t),\cdots,\theta^n(t)) + \gamma^{-1}(x,\theta(t),\cdots,\theta^{n-1}(t))v = -\dfrac{\hat{L}_f^n h(x,\theta(t))}{\overline{L}_g \hat{L}_f^{n-1} h(x,\theta(t))} + \dfrac{1}{\overline{L}_g \hat{L}_f^{n-1} h(x,\theta(t))} v,$ where $h(x,\theta(t))$ is satisfied with following partial derivative equations :

$$\overline{L}_{\overline{ad}_f^i g} h(x,\theta(t)) = 0, \quad i = 0,1,\cdots,n-2 \tag{3}$$

$$\overline{L}_g \hat{L}_f^{n-1} h(x,\theta(t)) \neq 0 \tag{4}$$

(2) becomes the following form :

$$\dot{z} = A_c z + B_c v \tag{5}$$

Lemma 1. If a function $\phi(x,\theta(t))$ solve the PDEs (3) and (4) by substituting $\phi(x,\theta(t))$ for $h(x,\theta(t))$ near x^0, then the function $\lambda(x,\theta(t))$ satisfies the PDEs if following conditions hold for some smooth function ψ in a neighborhood of x^0.

$$\lambda(x,\theta(t)) = \psi(\phi(x,\theta(t))) \tag{6}$$

$$\frac{\partial \psi}{\partial \phi} \neq 0 \tag{7}$$

$$\overline{L}_g \hat{L}_f^i \left(\frac{\partial \psi}{\partial \phi} \right) = 0 \quad (i = 0,1,\cdots,n-2) \tag{8}$$

By using $z = \hat{T}_i(x,\theta(t),\cdots,\theta^{n-1}(t))$, the system (1) is transformed into the following form :

$$\begin{aligned}
\dot{z} &= A_c z + B_c \gamma_i(x,\theta(t),\cdots,\theta^{n-1}(t))(u_i - \alpha_i(x,\theta(t),\cdots,\theta^n(t))) \\
&\quad + \frac{\partial \hat{T}_i}{\partial x} \Delta f(x,\theta(t)) + \frac{\partial \hat{T}_i}{\partial x} \Delta g(x,\theta(t)) u_i \qquad (9) \\
&= A_c z + B_c \gamma_i(x,\theta(t),\cdots,\theta^{n-1}(t))(u_i - \alpha_i(x,\theta(t),\cdots,\theta^n(t))) \\
&\quad + \Delta \overline{f}_i(z,\theta(t),\cdots,\theta^n(t)) + \Delta \overline{g}_i(z,\theta(t),\cdots,\theta^n(t)) u_i \qquad (10)
\end{aligned}$$

for $x \in D_x$, $z \in D_z = \hat{T}_i(D_x,\theta(t),\cdots,\theta^{n-1}(t))$ where $\{A_c, B_c\}$ is Brunovsky canonical pair, $\gamma_i(x,\theta(t),\cdots,\theta^{n-1}(t)) = \overline{L}_g \hat{L}_f^{n-1} h_i(x,\theta(t))$ and $\alpha_i(x,\theta(t),\cdots,\theta^n(t)) = -\frac{\hat{L}_f^n h_i(x,\theta(t))}{\overline{L}_g \hat{L}_f^{n-1} h_i(x,\theta(t))}$.

Assumption 2. There exist some constants $c_{\overline{fi}\_1}$, $c_{\overline{fi}\_2}$, $c_{\overline{gi}\_1}$, and $c_{\overline{gi}\_2}$ that satisfy following inequalities for $z \in D_z$.

$$\left\|\Delta\overline{f}_i(z,\theta(t),\cdots,\theta^n(t))\right\| \le C_{\overline{f}i\_1}\|z\| + C_{\overline{f}i\_2} \tag{11}$$

$$\left\|\Delta\overline{g}_i(z,\theta(t),\cdots,\theta^n(t))\right\| \le C_{\overline{g}i\_1}\|z\| + C_{\overline{g}i\_2} \tag{12}$$

Assumption 3. The uncertainties satisfy following conditions.

$$\Delta f(x,\theta(t)) = span\{g(x,\theta(t))\} = g(x,\theta(t)) f^*(x,\theta(t)) \tag{13}$$

$$\Delta g(x,\theta(t)) = span\{g(x,\theta(t))\} = g(x,\theta(t)) g^*(x,\theta(t)) \tag{14}$$

Lemma 2. If the assumption 2 and 3 are satisfied, the following inequalities are satisfied.

$$\left\|z^T P\Delta\overline{f}_i(z,t)\right\| \le (C_{\overline{f}i\_1}\|z\| + C_{\overline{f}i\_2})\left\|z^T PB_c\right\| \tag{15}$$

$$\left\|z^T P\Delta\overline{g}_i(z,t)\right\| \le (C_{\overline{g}i\_1}\|z\| + C_{\overline{g}i\_2})\left\|z^T PB_c\right\| \tag{16}$$

3 Controller Design

In this subsection, we propose a controller that compensates the uncertainties based on the Lyapunov redesign. We consider the following control structure of the form :

$$u_i = \alpha_i(x,\theta(t),\cdots,\theta^n(t)) + \gamma_i^{-1}(x,\theta(t),\cdots,\theta^{n-1}(t))v_i \tag{17}$$

$$v_i = v_{i\_1} + v_{i\_2} \tag{18}$$

where $v_{i\_2}(t)$ is a controller to overcome the uncertainties of the system.

Theorem 1. The uncertain nonlinear system (9) is asymptotically stable by using the control law given by

$$u_i = \alpha_i(x,\theta(t),\cdots,\theta^n(t)) + \gamma_i^{-1}(x,\theta(t),\cdots,\theta^{(n-1)}(t))(v_{i\_1} + v_{i\_2}) \tag{19}$$

$$v_{i\_1} = Kz = k_1 z_1 + \cdots + k_n z_n \tag{20}$$

$$v_{i\_2} = -\eta_i \frac{z^T PB_c}{\left\|z^T PB_c\right\|} \tag{21}$$

$$\eta_i = \frac{C_{\overline{f}i\_1}\|z\| + C_{\overline{f}i\_2} + \left(C_{\overline{g}i\_1}\|z\| + C_{\overline{g}i\_2}\right)\left(\|\alpha_i(x,t)\| + \left\|\gamma_i^{-1}(x,t)\right\|\|Kz\|\right)}{1 - \left(C_{\overline{g}i\_1}\|z\| + C_{\overline{g}i\_2}\right)\left\|\gamma_i^{-1}(x,t)\right\|} \tag{22}$$

for $z \in D_z^{'} = \{z \mid z \in D_z \ and\ \left(C_{\overline{g}i\_1}\|z\| + C_{\overline{g}i\_2}\right)\left\|\gamma_i^{-1}(x,t)\right\| < 1\}$ where $z = \hat{T}_i(x,t)$ is satisfied.

Proof. Let the Lyapunov function be $V = z^T Pz$ where $P > 0,\ Q > 0,\ A = A_c + B_c K$, and $A^T P + PA = -Q.$ By taking the derivative of V, we have $\dot{V} \le -\lambda_{\min}(Q)\|z\|^2$.

4 Switching Rule for Multi-diffeomorphism

The system given by (1) is transformed to (9) by $z = \hat{T}_i(x,t)$ and feedback controller u_i in Theorem 1, where $i = 1,2,\cdots,m.$ By the feedback controller u_i, the closed system is presented as

$$\dot{x} = f_i(x,t),\ i = 1,2,\cdots,m \tag{23}$$

Let $\dot{x} = f_{k_j}(x,t),\ k_j \in [1,m]$ and $j \in [0,\infty)$, denote the system that is active for $t_j \le t < t_{j+1}$.

Theorem 2. The origin of switched system (23) with the following switching rule is exponentially stable. Furthermore, the exponential upper bound of the switched system is smaller than that of the one system given by $\dot{x} = f_i(x,t),\ t \in [t_0,\infty)$.

-Switching rule :

Step 1 : Set $\mathrm{i} = 0$.

Step 2 : If

$$\min_{k_{i+1} \in \{2,\cdots,m\}} \frac{\alpha_{k_{i+1}}(x(t_i),t_i)}{\alpha_{k_{i+1}}(x(t),t)} < \left(\frac{M}{m}\right)^{-1} \frac{\alpha_1(x(t_i),t_i)}{\alpha_1(x(t),t)}$$

at $t = t_{i+1} > t_i$, then $\dot{x} = f_{k_{i+1}}(x,t)$ is active for $t \ge t_{i+1}$. Otherwise, $\dot{x} = f_1(x,t)$ is active for $t \ge t_{i+1}$.

Step 3 : Let $i = i+1$, go to Step 2.

Proof. For all t, $\|x(t)\| < \left(\frac{M}{m}\right)^{\frac{1}{2}} \frac{\alpha_1(x(t_0),t_0)}{\alpha_1(x(t),t)} \|x(t_0)\| e^{-\frac{\gamma}{2M}(t-t_0)}$

5 Conclusion

The aim of this paper is to propose a switching rule for uncertain time-varying nonlinear system by using multi-diffeomorphism. We formulate a set of problem mathematically in case of the existence of uncertainties. And then, the controllers and the switching rule for the switching control to improve transient behavior are proposed and analyzed in case of time-varying system.

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