

# Neuro-Adaptive Control of Nonlinear Dynamic System Using RBFN

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**Abstract.** Neural networks are known as kinds of intelligent strategies since they have learning capability. In this paper we propose an Adaptive Tracking Control of Nonlinear System the radial basis function network(RBFN) that is a kind of neural networks. The exploration versus exploitation dilemma of reinforcement learning is solved through smooth transitions between the two modes. The controller is capable of asymptotically approaching the desired reference trajectory, which is showed in simulation result.

**Keywords:** Radial Basis Function Network, Adaptive Tracking Control, Nonlinear System

## 1 Introduction

In this thesis, we propose a approach to NN-based self-tuning adaptive control, where the self-tuning principle and RL are combined, with special emphasis on resolving the exploration versus exploitation dilemma. The control scheme consists of a controller, a utility estimator, an exploration module, a learning module and a rewarding module[1]. The controller and the utility estimator are implemented together in a single Radial Basis Function neural network(RBFN)[2]. The learning method involves structural adaptation and parameter adaptation[3]. This enables rapid exploration response to novel plant dynamics and stable operation in an occurrence of changes in plant dynamics[4].

## 3 Formulation of nonlinear control problem

Assume that a single-input-single-output(SISO) nonlinear discrete system is given in the form

$$y(k+1)=f[q(k),u(k)] \text{ (state equation)} \tag{Eq(3.1)}$$

$$y(k)=g[q(k)] \text{ (output equation)}$$

Where  $q \in R^n$  are the state variables,  $u(k) \in R^1$  is the control input,  $f[.]$  and  $g[.]$  are the nonlinear maps on  $R^n$ ,  $f[.]$  is bounded away from zero, and  $y(k) \in R^1$

is the plant output. Finding a control signal  $u(k)$  that will force the output  $y(k)$  to track asymptotically the desired output  $y_d(k)$ , that is

$$\lim_{k \rightarrow \infty} [y_d(k) - y(k)] = 0 \quad \text{Eq(3.2)}$$

To achieve the above, the following assumptions about the nonlinear plant are required:

**Assumption 1.** for any  $q \in R^n$

$$0 < k_i \leq |f[\cdot]| \quad \text{Eq(3.3)}$$

**Assumption 2.** For any  $k \in [0, \infty]$ , the desired output  $y_d(k)$  and its n-derivatives  $y_d^{(1)}(k), y_d^{(2)}(k), \dots, y_d^{(n)}(k)$ , are uniformly bounded, that is,

$$|y_d^{(k)}(k)| \leq m_i \quad i = 0, 1, 2, \dots, n \quad \text{Eq(3.4)}$$

**Assumption 3.** There exist coefficients  $a_{ff}$  and  $b_{fb}$  such that  $\hat{f}[\cdot]$  and  $\hat{g}[\cdot]$  approximate the nonlinear functions  $f[\cdot]$  and  $g[\cdot]$ , respectively, with an accuracy  $\varepsilon$  on  $\Sigma$ , a compact subset of  $R^n$ , that is,

$$\begin{aligned} \max |f[\cdot] - \hat{f}[\cdot]| &\leq \varepsilon \\ \max |g[\cdot] - \hat{g}[\cdot]| &\leq \varepsilon \quad \forall q \in \text{on } \Sigma \end{aligned} \quad \text{Eq(3.5)}$$

A nonlinear system can be represented by one of the four discrete-time models as suggested in [2]. These models are described by the following nonlinear difference equations:

**Model I**

$$y(k+1) = \sum_{i=0}^{n-1} \alpha_i y(k-i) + g[u(k), u(k-1), \dots, u(k-m+1)] \quad \text{Eq(3.6)}$$

**Model II**

$$y(k+1) = f[y(k), y(k-1), \dots, y(k-n+1)] + \sum_{j=0}^{m-1} \beta_j u(k-j) \quad \text{Eq(3.7)}$$

**Model III**

$$y(k+1) = f[y(k), y(k-1), \dots, y(k-n+1)] + g[u(k), u(k-1), \dots, u(k-m+1)] \quad \text{Eq(3.8)}$$

**Model IV**

$$y(k+1) = f[y(k), y(k-1), \dots, y(k-n+1), u(k), u(k-1), \dots, u(k-m+1)] \quad \text{Eq(3.9)}$$

where  $[u(k), y(k)]$  represents the input-output pair of a SISO plant at time  $k$ . In all four models, the output of the plant at time  $(k+1)$  depends both upon its past  $n$  values as well as the past  $m$  values of the input (output of the neural net). In the above equations,  $f[\cdot]$  and  $g[\cdot]$  are nonlinear functions that may take different forms. In model I, the plant output  $y(k+1)$  is a linear function of the past values  $y(k-i)$ , while in Model II the relation between  $y(k+1)$  and the past values of the control input  $u(k-j)$  is assumed linear. In model III, the nonlinear relation of  $y(k+1)$  with  $y(k-i)$  and  $u(k-j)$  is

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assumed to be separable, while Model IV, in which  $y(k+1)$  is a nonlinear function of  $y(k-i)$  and  $u(k-j)$  subsumes Models I-III, is analytically least tractable.

The performance of the neural network with only synaptic adaptation has been discussed extensively for the above four models in [5]. A study of performance of the proposed neural network for a set of nonlinear systems represented by Model IV is reported in this thesis from the adaptive control point of view.

### 4 Radial Basis Function Neural Network (RBFN)

The reason is that RBFN allows to incrementally construct a function approximation without modifying the knowledge previously acquired when the weights are updated or new neurons are added to the network[6].

### 5 Simulation Examples

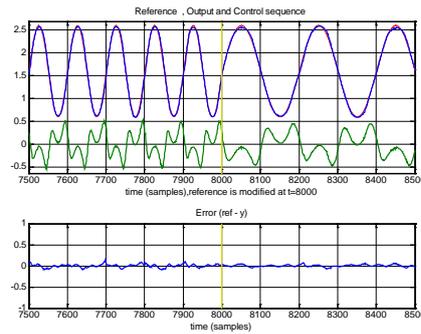


Figure 6.1 Input  $r(t)$  (dashed line), plant output  $y(t)$  and control signal  $u(t)$  are plotted for three time intervals: [7500:8500].

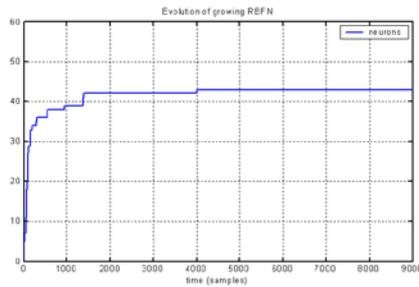


Figure 6.2 The number of neurons  $m$  is plotted during the evolution of growing RBFN.

## 6 Conclusion

Direct adaptive self-tuning neuro-control for time-dependent nonlinear plants has been proposed in this thesis. The controller and the utility estimator are implemented together in a single growing RBFN. Learning of the RBFN involves structural adaptation and parameter adaptation. The exploration–exploitation dilemma is resolved through smooth transitions between the two modes. This method has been shown to successfully control the simulated nonlinear system.

Of course, there are still some works need to improve in the future. In the study ,a simple exploration strategy was used, where only maximal amplitude of the perturbation was adapted on-line. A more elaborate exploration strategy could be utilized to improve the convergence properties.

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