A Flexible Cantilever Beam Control using a Tip Displacement Sensor

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Abstract. This paper shows an integral and derivative feedback controller for the control of the pointing error at the tip of a very flexible cantilever beam. A piezoelectric PZT actuator patch is bonded on near the fixed end and a displacement sensor is located at the tip of the beam. The beam is designed to be lightly damped and its step response without control is quite long. Computer simulation shows that the integral and derivative controller can be useful to diminish the pointing error with stability. After the integral and derivative control, the settling times become about 40 seconds when the gain margin is 0.9 dB and about 50 seconds when the gain margin is 6 dB. The integral-derivative control gives faster settling time and reduces overshoot response in the time domain.

Keywords: Integral feedback control, Lightly damped system, Non-minimum phase.

1 Introduction

A very lightly damped cantilever beam is considered and the objective is to minimize the pointing error after a sudden movement using the integral-derivative feedback controller. Lightly damped flexible structures have a rather long transient response once they are excited even if the excitation is very short [1-4]. The integral-derivative feedback controller is adopted to minimize the inherent long transient response of the flexible cantilever beam system. Since the sensor and the actuator are located apart, the non-minimum phase property of the plant can cause instability with a high gain [3-6]. In this paper, the integral-derivative controller will be discussed for this problem.

2 Flexible Beam and Piezocermic Patch Dynamics

A flexible cantilever beam with the damping ratio of $\zeta = 0.01$ is investigated and the beam has the fixed-free boundary condition. A piezoceramic PZT actuator is attached near the fixed end on the beam. By considering the boundary conditions of the beam,
the theoretical plant model for the deflection response at \( x = L_b \) to the moment at \( x = L_a \) due can be derived as [7-8]

\[
G(j\omega) = \frac{w(L_a)}{M(L_a)} = \sum_{n=1}^{\infty} \frac{\phi_n(L_a)\phi'_n(L_a)}{M_n[(\omega_n^2 - \omega^2) + j2\zeta_n\omega_n\omega]},
\]

where \( \zeta_n \) is the damping ratio and \( \phi'_n(L_a) \) is the spatial derivative of \( \phi_n(x) \) at \( x = L_a \). As an actuator, the piezoceramic PZT patch induces bending moment at the end of the patch [7].

3 Integral-Derivative Feedback Controller

As an analogue feedback controller, a combined form (integrator \( H_I \) + differentiator \( H_D \)) feedback controller \( H_{ID} \) is introduced for the problem. This controller can provide better steady-state and transient response as normal PID controllers can provide an acceptable degree of error reduction simultaneously with acceptable stability and damping. The inner loop can be regarded as \( G'(j\omega) \) which has \( H_D \) feedback controller, whereas the outer loop has \( H_I \) feedback controller. As the frequency response of the inner loop \( G'(j\omega) \) can be expressed as [6-9]

\[
G'(j\omega) = \frac{G(j\omega)}{1 + G(j\omega)H_D(j\omega)}
\]

The frequency responses of the open-loop and the closed-loop with the derivative feedback controller are expressed as

\[
\begin{pmatrix}
\frac{y(j\omega)}{r(j\omega)}
\end{pmatrix}_{OL} = G'(j\omega)H_I(j\omega)
\]

\[
\begin{pmatrix}
\frac{y(j\omega)}{r(j\omega)}
\end{pmatrix}_{CL} = \frac{G(j\omega)H_I(j\omega)}{1 + G(j\omega)(H_I(j\omega) + H_D(j\omega))}
\]
4 Control Simulation, Results and Discussions

For the computer simulation, a lightly damped beam with the length $L_b = 1200$ mm, the width $B_b = 30$ mm and the thickness $t_b = 2$ mm is considered. The PZT actuator is assumed 100 mm long ($L_a$), 30 mm wide, and 1 mm thick.

The integral-derivative controller showed a good performance than integral and derivative feedback controllers alone with the same 6dB gain margin. The gains which produced the best response were determined empirically to be $K_I = 0.115$, $K_D = 0.001$ ($\zeta = 0.01$). As can be seen from Figure 1(a), the settling time to a step input before control is about 70 seconds. After the integral and derivative control, the settling times become about 40 seconds when the gain margin is 0.9 dB as plotted in Figure 1(b) and about 50 seconds when the gain margin is 6 dB as illustrated in Figure 1(c). The integral-derivative control gives faster settling time and reduces overshoot response in the time domain.

Fig. 1. Step responses before and after control of the flexible cantilever beam system in the time domain. (a) Before control: about 70 seconds. (b) After control: about 40 seconds when the gain margin is 0.9 dB. (c) After control: about 50 seconds when the gain margin is 6 dB.
5 Conclusions

This paper shows an integral and derivative feedback controller for the control of the pointing error at the tip of a very flexible cantilever beam. The beam is designed to be lightly damped and its step response without control is quite long. Computer simulation shows that the integral and derivative controller can be useful to diminish the pointing error with stability. After the integral and derivative control, the settling times become about 40 seconds when the gain margin is 0.9 dB and about 50 seconds when the gain margin is 6 dB. The integral-derivative control gives faster settling time and reduces overshoot response in the time domain.

References

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