Longitudinal Motion Characteristics between a Non-Matched Piezoelectric Sensor and Actuator Pair

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Abstract. This paper shows theoretical input and output relationship between a non-matched piezoelectric sensor and actuator pair which is attached on an infinite beam in terms of longitudinal wave propagation. The results show that the non-matchness of the pair causes a larger phase change as the distance between the sensor and actuator increases more. However the magnitude and phase responses in this case provide a better understanding in designing a vibration control system.

Keywords: Longitudinal Response, Partly Overlapped Sensor and Actuator, Infinite Beam

1 Introduction

A piezoelectric actuator and sensor pair has been widely considered as an alternative to a conventional actuator and sensor pair to control light and flexible structures [1-3]. The location of a sensor and actuator pair is an essential problem for controlling the waves on structures [2-4]. The input and output relationship between the piezoelectric sensor and actuator pair which is non-matched is quite different from the fully matched sensor and actuator pair. In this investigation, the longitudinal motion characteristics between the non-matched piezoelectric sensor and actuator pair are examined theoretically.

2 Non-Matched Longitudinal Piezoelectric Sensing and Actuation

2.1 Longitudinal piezoelectric actuation model

Theoretical actuation modeling of a non-matched piezoelectric actuator attached on a side of an infinite beam is investigated. The thickness of bonding layer between the beam and the actuator is ignored. It is known that the effective longitudinal force due to the actuator can be defined as
\[ f = Y_c A_c \varepsilon_c = Y_c A_o \Delta_o \Lambda \]  \hspace{1cm} (1)

where \( Y_c \), \( I_c \), \( A \) and \( \Lambda \) are Young’s modulus, 2nd moment, bending strain and the free piezoelectric strain \( \Lambda = d_{31}E_3 = d_{31}V_3 / h_p \), respectively. In which \( d_{31} \) is the piezoelectric strain constant, \( E_3 \) is the electric field strength in the direction 3 and \( h_p \) the thickness of the piezo actuator or sensor. Also, the thickness ratio is defined by \( T = h_p / h_b \), where \( h_p \) and \( h_b \) are the thickness of the piezo transducer and the beam.

2.2 Longitudinal waves on an infinite beam

The analytical longitudinal wave model due to the excitation of a piezoelectric actuator patch (PZT, \( L_p \times B \times h_p = 50 \times 30 \times 1 \text{ mm} \)) attached on an infinite beam (aluminium, \( L_b \times B \times h_b = \infty \times 30 \times 6.35 \text{ mm} \)) is considered.

As shown in Figure 1, the infinite beam has four different longitudinal waves induced by the piezoelectric actuator patch and it is assumed there are no reflected waves. A piezoelectric sensor (PZT, \( L_p \times B \times h_p = 50 \times 30 \times 1 \text{ mm} \)) is attached without any matchness on the other side of the beam and has the same dimension of the actuator.

![Fig. 1 Four different longitudinal waves on an infinite beam due to a piezo actuator](image)

Since the two compressive longitudinal forces are \( F = -F_A = F_B \), the amplitudes of the four longitudinal waves have the following relationship as \( A_{o1} = A_{o2} = -B_{o1} = B_{o2} \), where the amplitude of the wave \( A_{o2} = jF / 2Y_c A_h k_o \).
2.3 Sensing Longitudinal waves on the infinite beam

The piezoelectric sensor is positioned between $x = x_1 > L_p$ and $x = x_1 + L_p$ as illustrated in Figure 1, however the piezoelectric actuator is bonded on the beam between $x = 0$ and $x = L_p$. So there is no matched part at all between the two transducers on the beam. As shown in Figure 1, the non-matched piezoelectric sensor only detects the two longitudinal waves: two right-going waves $A_{o2}$ and $B_{o2}$. Thus, the longitudinal displacement $u(x)$ in the region of $x > L_p$, as shown in Figure 1, can be expressed by the two longitudinal waves

$$u(x) = A_{o2}e^{-jk_2x} + B_{o2}e^{-jk_2(x-L_p)}$$  \hspace{1cm} (2)$$

The electric charge output of the piezoelectric sensor in this case can be described by [2]

$$q_o(t) = \int_{x_i}^{x_f} e_{31}B \frac{du}{dx} \, dx = e_{31}B[u(x_f) - u(x_i)]$$ \hspace{1cm} (3)$$

where $e_{31}$ is the piezoelectric stress constant. Therefore the transfer function between the charge output from the piezoelectric sensor against the applied electric field $E_3$ to the piezoelectric actuator in terms of the longitudinal motion can be obtained.

3 Analysis and Discussion of Longitudinal Response

The longitudinal wave actuation and sensing model described in the previous section has been analyzed using a computer simulation. The four different distances between the sensor and actuator are designated as $x_1 = 1.03L_p$, $x_1 = 1.37L_p$, $x_1 = 1.61L_p$, and $x_1 = 1.97L_p$ when 100 V is applied to the piezoelectric actuator. The frequency response functions of each distance are plotted in Figure 1.

The results show that the non-matchness of the pair causes a larger phase change as the distance between the sensor and actuator increases more. However the magnitude and phase responses in this case provide a better understanding in designing a vibration control system.

4 Conclusions

This paper shows theoretical input and output relationship between a non-matched piezoelectric sensor and actuator pair which is attached on an infinite beam in terms of longitudinal wave propagation. The results show that the non-matchness of the pair causes a larger phase change as the distance between the sensor and actuator increases.
more. However the magnitude and phase responses in this case provide a better understanding in designing a vibration control system.

Fig. 2 Calculated input-output relationship w.r.t. frequency of the piezoelectric sensor output against the longitudinal force when the applied voltage to the piezoelectric actuator is 100 V. Solid line: $x_1 = 1.03L_p$. Dashed line: $x_1 = 1.37L_p$. Dashed and dotted line: $x_1 = 1.61L_p$. Dotted line: $x_1 = 1.97L_p$.

References