Research on Efficient Turbo Frequency Domain Equalization in STBC-MIMO System

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Abstract. An efficient Turbo Frequency Domain Equalization (FDE) based on symbol-wise minimum mean-square error (MMSE) filtering is proposed for a novel space-time block code (STBC) MIMO system. The transmitter sends a separate data block via STBC using two antennas per group to get diversity gain. The receiver can effectively utilize inter-antenna interference (IAI) and inter-symbol interference (ISI) followed by frequency domain equalization to process soft interference cancellation (SIC). After frequency domain filtering, the symbol Log-likelihood ratio (LLRs) calculated from the outputs of equalizer is as the inputs of the soft-in soft-out (SISO) decoder. Simulation results show that our proposed scheme provides a further substantial gain while not increasing complexity at the receiver.

Keywords: MIMO; MMSE; single carrier; STBC; Turbo FDE.

1 Introduction

In this paper, we develop an iterative detection and decoding algorithm of SC-FDE based on [1] for a novel STBCMIMO wireless system. The receiver can effectively utilize inter-antenna interference (IAI) and inter-symbol interference (ISI) followed by frequency domain equalization to process soft interference cancellation(SIC), and symbol Log-likelihood ratio (LLR) is calculated as the inputs of the soft-in soft-out (SISO) decoder using the outputs of equalizer. So it can realize iterative channel equalization and channel decoding at each iteration. Theory analysis and simulation results both show that our proposed algorithm can improve the system performance remarkably compared with general MIMO system.

2 System Overview

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We define that $x_p^{(t)}$ (p =1, 2..., P ; t = v, v + 1; v =0, 2, ...) is the t^{th} block signal on the p^{th} stream before space-time block coding. After transferring each signal block into frequency domain by N-point Fast Fourier Transform (FFT), corresponding signal is $X_p^{(t)}.x_{p_i}^{(t)}(i=1,2)$ is the t^{th} block signal on p_i^{th} transmitted antenna(i^{th} antenna on the p^{th} stream) after being encoded according to STBC principle. Where,

$$\begin{cases} x_p^{(t)} = \left(x_p^{(t)}(1), x_p^{(t)}(2), \ldots x_p^{(t)}(N)\right)^T \\ X_{p_i}^{(t)} = \left(X_{p_i}^{(t)}(1), X_{p_i}^{(t)}(2), \ldots X_{p_i}^{(t)}(N)\right)^T \\ x_{p_i}^{(t)} = \left(x_{p_i}^{(t)}(1), x_{p_i}^{(t)}(2), \ldots x_{p_i}^{(t)}(N)\right)^T \end{cases} \quad (1)$$

Block-wise STBC principle is shown as

$$\begin{cases} x_{p_1}^{(t)}(n) = x_p^{(t)}(n), x_{p_1}^{(t+1)}(n) = -x_p^{(t+1)^*}(n) \\ x_{p_2}^{(t)}(n) = x_p^{(t+1)}(n), x_{p_2}^{(t+1)}(n) = x_p^{(t)^*}(n) \end{cases} \quad (2)$$

At the receiver, after discarding the CP, the time domain signals on q^{th} antenna, for the t^{th} block can be expressed as $r_q^{(t)}$ (q=1,2,…,Q),and the corresponding received frequency domain signal is $R_q^{(t)}$ .Where,

$$\begin{cases} r_q^{(t)} = \left(r_q^{(t)}(1), r_q^{(t)}(2), \ldots r_q^{(t)}(N)\right)^T \\ R_q^{(t)} = \left(R_q^{(t)}(1), R_q^{(t)}(2), \ldots R_q^{(t)}(N)\right)^T \end{cases} \quad (3)$$

In order to express clearly ,we define

$$\begin{cases} x = \left[x_1^{(t)^T}, x_1^{(t+1)^T}, \ldots, x_P^{(t)^T}, x_P^{(t+1)^T}\right]^T \\ X(k) = \left(X_1^{(t)}(k), X_1^{(t+1)}(k), \ldots, X_P^{(t)}(k), X_P^{(t+1)}(k)\right)^T \\ R(k) = \left(R_1^{(t)}(k), R_1^{(t+1)^*}(k), \ldots, R_Q^{(t)}(k), R_Q^{(t+1)^*}(k)\right)^T \\ k = 1, 2 \ldots N \end{cases} \quad (4)$$

Let $E_k = [e_k, e_k, \cdots, e_k]$, where e_k denotes the unit vector that can get k^{th} frequency domain signal at the k^{th} (k=0,…N-1) tone can be written as,

$$R(k) = H(k) E_k^H \tilde{F}_{2P} x + Z(k) = H(k) X(k) + Z(k) \quad (5)$$

Where,

$$H(k) = \begin{pmatrix} H_{1,1_1}(k) & H_{1,1_2}(k) & \cdots & H_{1,P_1}(k) & H_{1,P_2}(k) \\ H_{1,1_2}^*(k) & -H_{1,1_1}^*(k) & \cdots & H_{1,P_2}^*(k) & -H_{1,P_1}^*(k) \\ \vdots & \vdots & & \vdots & \vdots \\ H_{Q,1_1}(k) & H_{Q,1_2}(k) & \cdots & H_{Q,P_1}(k) & H_{Q,P_2}(k) \\ H_{Q,1_2}^*(k) & -H_{Q,1_1}^*(k) & \cdots & H_{Q,P_2}^*(k) & -H_{Q,P_1}^*(k) \end{pmatrix}$$

3 Frequency Domain MMSE Turbo Equalization

We describe $x = \left[x_1^{(t)^T}, x_1^{(t+1)^T}, \ldots, x_P^{(t)^T}, x_P^{(t+1)^T}\right]^T = \left[x_1^T, x_2^T, \ldots, x_{2P}^T\right]^T$

A. Soft ISI and IAI Cancellation

The estimate of the desired symbol can be produced by a frequency domain MMSE filter after the ISI and IAI cancellation in the frequency domain. We assume the n^{th} symbol on the i^{th} ith antenna $x_i(n)$ (i =1,2, ...2P ; n =1, 2, ...N) is the desired symbol.

In the paper, the expected IAI and ISI for $x_i(n)$ can be presented as $\alpha_i(k)$ and $\beta_{i,n}(k)$ respectively, $\alpha_i(k) = H(k) E_k^H \tilde{F}_{2P} x_{\hat{i}}$ **(6)**

$$\beta_{i,n}(k) = H(k) E_k^H \tilde{F}_{2P} x_{\hat{i},\hat{n}} \quad \textbf{(7)}$$

Where,
$$\begin{cases} x_{\hat{i}} = \left[\overline{x}_1^T, \cdots, \overline{x}_{i-1}^T, 0^T, \overline{x}_{i+1}^T, \cdots, x_{2P}^T\right]^T \\ x_{\hat{i},\hat{n}} = \left[0^T, \cdots, 0^T, \hat{x}_{\hat{i},\hat{n}}^T, 0^T, 0^T, \cdots 0^T\right]^T \\ \hat{x}_{\hat{i},\hat{n}} = \left[\overline{x}_i(1), \cdots \overline{x}_i(n-1), 0, \overline{x}_i(n+1), \cdots \overline{x}_i(N)\right]^T \end{cases} \quad \textbf{(8)}$$

after SIC, the signal in the frequency domain at k^{th} tone is written as,

$$\begin{aligned} Y_{i,n}(k) &= R(k) - \alpha_i(k) - \beta_{i,n}(k) = H(k) E_k^H \tilde{F}_{2P}\left(x - x_{\hat{i}} - x_{\hat{i},\hat{n}}\right) + Z(k) \\ &= H(k) E_k^H \tilde{F}_{2P}\left(x - \hat{x}_{i,n}\right) + Z(k) \end{aligned} \quad \textbf{(9)}$$

Considering all frequency tones, the soft interference cancellation model can be expressed as follows,

$$Y_{i,n} = R - \alpha_i - \beta_{i,n} = H\tilde{F}_{2P}\left(x - \hat{x}_{i,n}\right) + Z \quad \textbf{(10)}$$

After soft interference cancellation, signal to interference plus noise ratio (SINR) of signal $Y_{i,n}$ has been improved compared with original received data. Then frequency domain MMSE equalization is implemented, while soft interference is ignored in iteration zero since there is no priori information.

B. Efficient Frequency Domain MMSE Filtering

Symbol-wise MMSE criterion can be written as (11) in order to detect the desired symbol $x_i(n)$. $D_{i,n} = \arg_{D_{i,n}}^{\min} E\left\{\left|D_{i,n} Y_{i,n} - x_i(n)\right|^2\right\}$ **(11)**

According to the orthogonality principle, we have $E\left\{\left(D_{i,n} Y_{i,n} - x_i(n)\right) Y_{i,n}^H\right\} = 0$ **(12)**

Then (10) is substituted into (11) with an assumption that there is no correlation between data symbols and AWGN.

$$D_{i,n} H\tilde{F}_{2P} E\left\{\left(x - \hat{x}_{i,n}\right)\left(x - \hat{x}_{i,n}\right)^H\right\} \tilde{F}_{2P} H^H + D_{i,n} E\left\{ZZ^H\right\} - E\left\{x_i(n)\left(x - \hat{x}_{i,n}\right)\right\} \tilde{F}_{2P}^H H^H = 0 \quad \textbf{(13)}$$

We assumed that the symbols are independent, and the priori information about the desired symbol $x_i(n)$ should not be used in the evaluation, then we have,

$$E\left\{x_i(n)\left(x - \hat{x}_{i,n}\right)^H\right\} = \Phi_{i,n} = \left[\underbrace{0, \cdots, 0}_{(i-1)N=n-1}, \sigma_s^2, \underbrace{0, \cdots, 0}_{(2P-i+1)N-n}\right] = \sigma_s^2 e_{i,n}^H \quad \textbf{(14)}$$

$$E\left\{\left(x - \hat{x}_{i,n}\right)\left(x - \hat{x}_{i,n}\right)^H\right\} = \Gamma_{i,n} = diag\left\{\Upsilon_1, \Upsilon_2, \cdots, \Upsilon_{i-1}, \Upsilon_i, \Upsilon_{i+1}, \Upsilon_{2P}\right\} \quad \textbf{(15)}$$

where $\Upsilon_j = diag\left\{\Upsilon_j^2(1), \Upsilon_j^2(2), \cdots, \Upsilon_j^2(N)\right\} (j \neq i)$, and $\Upsilon_j^2(m)$ is variance of symbol xj on the basis of prior information from decoder. $\tau_i = diag\left\{\Upsilon_i^2(1), \Upsilon_i^2(2), \cdots, \Upsilon_i^2(n-1), \sigma_s^2, \Upsilon_i^2(n+1), \cdots \Upsilon_i^2(N)\right\}$ and σ_s^2 is the symbol energy.

$$\Upsilon_j^2(m) = \mathrm{cov}\left(x_j^m, x_j^m\right) \qquad \textbf{(16)}$$

Finally, we obtained $$D_{i,n} = \Phi_{i,n}\tilde{F}_{2P}^H H^H\left(H\tilde{F}_{2P}\Gamma_{i,n}\tilde{F}_{2P}^H H^H + \sigma_s^2 I_{2Q*N}\right)^{-1} \qquad \textbf{(17)}$$

C. Extrinsic LLR Calculation

After equalization, the estimate of time domain symbol xi(n) can be obtained by IFFT,

$$\hat{x}_i(n) = D_{i,n}Y_{i,n} = D_{i,n}\left(H\tilde{F}_{2P}\left(x - \hat{x}_{i,n}\right) + Z\right) = D_{i,n}\left(H\tilde{F}_{2P}\left(e_{i,n}x_i(n) + \hat{x}'_{i,n} - \hat{x}_{i,n}\right)\right)D_{i,n}Z \qquad \textbf{(18)}$$

In (18), we can see that the first term is the expected symbol multiplied by a factor, the second term is the residual interference from other antennas and symbols, the third term is AWGN. As the iteration continue, the prior information becomes more and more exact. So an assumption is made in [9] that the output of MMSE equalizer has undergone a equivalent Gaussian channel,

$$\hat{x}_i(n) = \phi_i(n)x_i(n) + \lambda_i(n) \qquad \textbf{(19)}$$

Then the soft-input soft-output decoder can utilize extrinsic information from equalizer which is treated as the prior information to calculate extrinsic LLR according to the expectation and variance of equalized data symbol. As described in [3], expectation $\mu_i(n)$ $x_i(n)$ can be computed,

$$\mu_i(n) = D_{i,n}H\tilde{F}_{2P}e_{i,n}\sigma_s^2 \qquad \textbf{(20)}$$

Variance $\sigma_i^2(n)$ is written as $\sigma_i^2(n) = \left|renew\left(\hat{x}_i(n)\right)\mu_i(n) - \hat{x}_i(n)\right|^2$ **(21)**

The function of renew(.) is to renew $x_i(n)$ as a original modulated symbol. Then extrinsic LLR for the symbol can be obtained,

$$Le\left(x_i(n)\right) = \frac{2\hat{x}_i(n)\mu_i(n)}{\sigma_i^2(n)} \qquad \textbf{(22)}$$

D. Low Complexity Implementation

As the equalization is processed based on symbol-wise, it's hard to implement due to the complexity. Considering that the diagonal elements of the frequency domain covariance matrix $\Theta_{i,n} = \tilde{F}_{2P}\Gamma_{i,n}\tilde{F}_{2P}^H$ in (17) are constant with the same value,

$$\omega_j = \frac{1}{N}\left[\sum_{n=1}^{N}\gamma_j^2(n)\right](j \neq i) \qquad \textbf{(23)}$$

The off-diagonal elements can be ignored, because the diagonal elements is larger than the off-diagonal elements. Therefore, we approximate,

$$\omega_i = \frac{1}{N}\left[\sum_{n=1}^{N}\gamma_i^2(n)\right] \qquad \textbf{(24)}$$

After simplication, $\Theta_{i,n} = diag\left\{\omega_1 I_N, \cdots \omega_{i-1} I_N, \nu I_N, \omega_{i+1} I_N, \cdots \omega_N I_N\right\}$ **(26)**

Accordingly equalizer coefficients are given by,

$D_{i,n} = \Phi_{i,n}\tilde{F}_{2P}^H H^H \left(H\Theta_i H^H + \sigma_s^2 I_{2Q*N}\right)^{-1}$ **(27)**

4 Simulation Results

The BER performance of our proposed turbo equalization algorithm for STBC-MIMO system is showed in Fig.1. It is obvious that our proposed iterative equalizer achieves significant performance compared with the traditional non-iterative ones, especially under well channel condition. As the iteration times increases, the performance of the proposed system is better, but the iterative gains become comparatively smaller, especially after 3 iterations.

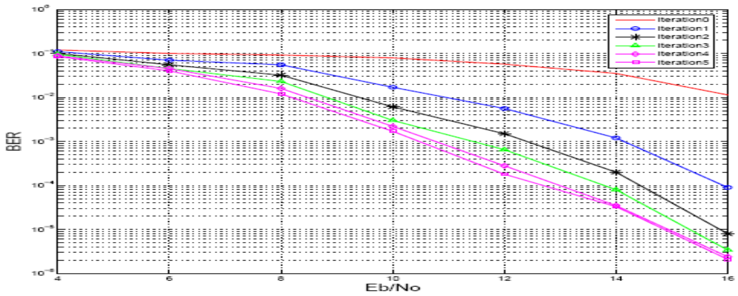

Fig. 1. BER performance of our proposed equalizer for STBC-MIMO

5 Conclusion

In this paper, we propose a novel Turbo FDE based on symbol-wise detection for single carrier STBC-MIMO system. The transmitter antennas double to get diversity gain without increasing receiving antennas. This algorithm can effectively utilize inter-antenna interference(IAI) and inter-symbol interference(IAI) followed by frequency domain equalization to process soft interference cancellation(SIC). Simulation results have shown that our proposed algorithm achieves better BER performance compared to both the traditional, non-iterative ones, and ones with general MIMO system.

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