A Study of Estimation Methods for Reliability of Common Cause Failure at k-out of-n System

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Abstract. System lifespan modeling becomes more complicated when an external event causes the succession of failure in 2 or more system components. Models that don’t consider this common cause failure cause the wrong estimation of system reliability. Let us assume that there are data composed of numbers of successive failures in i, i = 1, ..., n out of n components of system. Data having these characteristics are closely related with the number of system components. This is a problem occurring frequently in the estimation of reliability with respect to environmental change among components in various systems. This paper is intended to examine the estimation methods of reliability for common cause failure at k-out of-n system.

Keywords: binomial failure model, common cause failure, k-out of-n system, Poisson Process, reliability

1 Introduction

In the assessment of system reliability, it is possible to make a model using as variable its lifespan together with addition external event where one or more components sharing a potential cause of failure become the cause of system failure. A model made like this is called a model for common cause failure. Many components dependent on each other are necessary to the operation of system in order to maintain high reliability on the whole in industries requiring high reliability, such as aircraft or nuclear power industries. The advantage of using many components dependent on each other in system configuration is to prevent the operation of system from being stopped by a single event occurring by chance. Accordingly, it is an important factor that can reduce the loss of reliability in this system.

For example, in a nuclear power plant, the advantage reduces the risk from a major external accident occurring by chance, using 2 or more standby generators that can supply electric power. If a failure occurs in a generator, a standby diesel generator operates and generates electricity until the main generator works. The second generator located in the same place as that of the first generator operates only when the main diesel generator fails.
There are many research papers on a study of common cause failure in system reliability or its modeling. There are various fields having room for applying the model of common cause failure. And Billington et al.(1981) gave an example of transmission line system for transporting electrical power from power plant or substation to other power plant or substation or integration. Watson(1985) wrote that a model of common cause failure could be applied to the calculation of reliability of parallel-connected aircraft engine as well.

A group of components sharing a common cause failure event is called a common cause group. A problem may occur if the size of real group where data can be obtained is different from that of group necessary for study when the size of common cause group is fixed in certain common cause failure models. The work of mediating a difference between two groups is called mapping, and is generally used for a study on reliability of nuclear power plant. This study is intended to examine the estimation methods of reliability in case a common cause failure occurs in k-out of-n system.

2 Basic Concepts of CCF Modeling

Conventional common cause failure analysis model can be classified into shock model and nonshock model. The nonshock model is a model that analyzes a common cause failure, using conditional probability, and includes typical models such as basic parameter model suggested by Mosleh(1991), BF(beta factor) model suggested by Fleming(1975) and AF(alpha factor) model suggested by Mosleh et al.(1987). The above-mentioned suggested models are based on binomial failure model with regard to the reliability and failure rate of common cause failure occurring in case most of systems are connected in series or in parallel. And the reliability of k-out of-n system composed of parallel-series circuits remains a subject of study.

2.1 Common Cause Failures (CCF)

Nuclear power plant is a complex technological system which requires high level of operational safety and reliability. Safety systems include motor-operated valves (MOV), emergency diesel generators (EDG), water pumps, power batteries and many other devices depending on the functional requirement. In order to achieve high reliability, redundancy of various orders is added to safety systems. Most of the common causes can be classified into four types, namely, hardware equipment failure, human error during operation, environmental stress applied to components, and external events that causes stress (Mosleh et al., 1989). The key objective of this Section is to introduce the basic concepts and terminology associated with the modeling of common cause failures.

2.2 Poisson Process

The probabilistic basis for CCF modeling is that the occurrences of failures in a single component are modeled as the homogeneous Poisson process (PP). It means:
(1) Failures are purely random without any trend due to ageing,
(2) Occurrences of failure events are independent of each other, and
(3) After a failure component is renewed to its original “as new” condition.
In a time interval \((0, t)\), the number of failures are given by the Poisson distribution as

\[
P[N(t) = k] = \frac{(\lambda t)^k}{k!} e^{-\lambda t}
\]

Note that the parameter \(\lambda\) denotes the failure rate, defined as the average number of failures per unit time. The reliability, i.e., no failure in a time interval \((0, t)\) is given

\[
R(t) = P[N(t) = 0] = e^{-\lambda t}
\]

Thus, the failure rate is a single parameter that determines the component reliability. Based on component reliabilities, the system reliability can be evaluated.

### 2.3 \(k\)-out of-\(n\) system

Historically, several conceptual probabilistic models of CCF events have been presented in the literature. The \(k\)-out of-\(n\) system has been a new and widely accepted model of CCF events. The basic idea is that a failure observed at the system level involving either a single component failure or a failure of \(k\)-components is caused by an external shock generated by independent PPs. If at least \(k\) or more components are operating in a system composed of \(n\) components \(C_1, C_2, \ldots, C_n\), this system is called \(k\)-out of-\(n\) system.

In short, \(n\) independent PPs are generating external shocks that cause CCF events of various multiplicities. These PPs are mutually exclusive, i.e., one PP is in action at any given time.

In an \(n\)-component system, failure event data are described using the following parameters \((k=1, 2, \ldots, n)\)

- \(N_{k/n}\) = Number of failures involving any \(k\) components
- \(T\) = Operation time of the system in which \(N_{k/n}\) failures occurred
- \(\Lambda_{k/n}\) = Failure rate of the HPP causing \(k\) out of \(n\) component failures
- \(\Lambda_n\) = Sum of failure rates of all \(n\) HPP = \(\sum_{k=1}^{n} \Lambda_{k/n}\)

The failure events that are observed at the system level are caused by component failures. So it is assumed that component failures are caused by shocks modeled as PPs. Since component failures produce system failure events, the failure rates at the system and component levels are related. Define

\[
\lambda_{k/n} = \text{Failure rate of an PP causing a CCF involving } k \text{ specific components}
\]

The failure rates at the system and component levels are then related as
Let's define a one-out-of-three system, which means at least one component should be reliable in order to assure that the system is reliable. The failure of an $i^{th}$ component is modeled as an PP with the failure rate $\lambda_i$. The cut set of the system failure shown below:

$$P_s(t) = P_1 P_2 P_3 = (1 - e^{-\lambda_1T})(1 - e^{-\lambda_2T})(1 - e^{-\lambda_3T})$$

**1) in case of 1-out-of-3 system**

In the case of 1-out-of-3 system, which means at least one component should be reliable in order to assure that the system is reliable. The failure of an $i^{th}$ component is modeled as an PP with the failure rate $\lambda_i$. The cut set of the system failure shown below:

$$\Lambda_k/n = \binom{n}{k} \lambda_k/n = \frac{N_k/n}{T}$$

$$\Lambda_n = \sum_{k=1}^{n} \binom{n}{k} \lambda_k/n = \sum_{k=1}^{n} \Lambda_k/n$$

**Fig. 1.** Example of k-out of n system (in case of 2-out of 3 system)

First consider that the system involves only independent failures of the components. In order to fail the system, all the three components must fail. Thus, the probability of failure of the system is

$$P_s(t) = P_1 P_2 P_3 = (1 - e^{-\lambda_1T})(1 - e^{-\lambda_2T})(1 - e^{-\lambda_3T})$$

**2) in case of 2-out-of-3 system**

In the case of 2-out-of-3 system, parallel circuit of series systems. The failure of an $i^{th}$ component is modeled as an PP with the failure rate $\lambda_i$. And let's assume that $C_i$, $i = 1, \ldots, n$ is independent from each other. And, shows an example of parallel circuit of series systems, that is to say, 2-out-of-3 system of parallel-series circuit.

**Fig. 2.** Example of k-out of n system (in case of 2-out of 3 system)
This system becomes the operating status, if 2 or more components out of 3 components, i.e., $C_1$, $C_2$, and $C_3$, are operating. If all of $n$ components have the same reliability $\lambda_i = \lambda$, $i = 1, ..., n$, the probability that $k$-out of-$n$ system operates is as follows.

$$ P( k\text{-out of-}n \text{ system operating} ) = \sum_{m=k}^{n} \binom{n}{m} \lambda^m (1 - \lambda)^{n-m} \quad (2-1) $$

Where $m$ is the number of components that are operating.

5 Conclusions

This paper can estimate the method of estimating the reliability of system while a system is composed of parallel-series circuit or series-parallel circuit in case of using the estimation method of reliability in binomial failure model suggested by Marshall et al. (1967), various estimation methods studied after that time, and equation (2-1).

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