System Identification of the Quadrotor Flying Robot in Hover using Prediction Error Method

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Abstract. In this paper, a practical system identification method is applied to obtain a linear time-invariant (LTI) model of a quadrotor using input-output flight data as opposed to painstaking mathematical modeling techniques using the prediction error method with the black-box model approach. The fidelity verifications show that the proposed method provides a fairly accurate model of the quadrotor flying robot. This acquisition of high fidelity system model of the quadrotor will be a crucial step for the successful design of high-performance flight control system.

Keywords: System Identification, Prediction Error method, Quadrotor Flying Robot, Unmanned Aerial Vehicle (UAV)

1 Introduction

In recent years, the quadrotor flying robot that is a standard platform in the experiments and applications of mini unmanned aerial vehicles (mini-UAV). However, there has been relatively little development of accurate dynamic models of quadrotors. The acquisition of high fidelity system models and control techniques based upon the models are critical for precision control. In general, however, it is a challenging process to identify dynamics due to its multi-input multi-output (MIMO) characteristics, nonlinearity, several noise and disturbance. Hence, it has been often attempted to design the flight controllers of simple structure by trial and error during the actual flight. This alternative approach would require only the basic understanding of the plant, but the resulting controller by this process is not likely to be the optimal tuning.

Therefore, in this paper, we adopt a practical system identification method to obtain a linear time-invariant (LTI) model of a quadrotor using input-output actual flight data as opposed to painstaking mathematical modeling techniques. The commercial off-the-shelf quadrotor platform that has an embedded inner-loop attitude feedback controller was selected for this research. The system response data from the platform is acquired in carefully devised experiment procedure and then an LTI model is obtained using the prediction error method with the black-box model approach [1]. The fidelity verifications show that the proposed method provides a fairly accurate model of the quadrotor flying robot. This acquisition of high fidelity system model of the quadrotor will be a crucial step for the successful design of mid-
loop velocity feedback control and the outer-loop position control in our further studies.

2 Identification of Dynamic Model

The system identification of a dynamic system consists of the several steps as shown in Fig. 1.

The first step is the design of experiment to acquire the input/output data over a time interval. Acquiring data is not trivial and could be very laborious and expensive. The second step defines the structure of the system, for example, type and the order of the differential equation relating the input to the output. The third step is identification /estimation, which involves determining the numerical values of the structural parameters, which minimize the error between the system to be identified, and its model. The final step, validation, consists of relating the system to the identified model responses in time or frequency domain to instill confidence in the obtained model [2]. If identified model is not accepted, some more complex model structure must be considered, its parameters estimated, and the new model validated [3].

2.1 Experimental Setup

The aerial platform (Hummingbird) used in this study is a quadrotor developed by Ascending Technologies GmbH [4]. This platform is stabilized the three rotational degrees of freedom basically with PD controllers. Thus, no direct input signals to the rotors are needed; instead the desired tilt angle, yaw rate, and total thrust can directly be transmitted. The quadrotor is manually controlled near hover flight in an indoor space to avoid disturbances like the wind. The real time computer (RTC) from dSPACE GmbH [5] is used to log flight data and input commands from RC transmitter with 10Hz sampling frequency. The experimental setup for this process is illustrated in Fig. 2.
2.2 Characterization of Model Structure

Due to the quadrotor’s small size and symmetry, it can be modeled as a point-mass [6]. According to the principle of linear momentum (Newton’s 2nd Law) and the assumption that the mass $m$ of the quadrotor is constant and then the change of the momentum follows

$$m\ddot{r}_i = \sum F_i$$  \hspace{1cm} (1)

where $\ddot{r}_i$ is the acceleration of each of the Cartesian axes at the center of gravity, $F_i$ represents any outer force acting on the point-mass, and $I$ represents the inertial frame. The equation of motion of the quadrotor follows

$$\ddot{r}_i = \frac{1}{m} R_{IB}(\phi, \theta, \psi) \begin{bmatrix} 0 \\ 0 \\ -T \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}$$ \hspace{1cm} (2)

where $T$ represents the total thrust, pointing in the opposing direction of the quadrotor’s down axis, and $g$ is acceleration of gravity. $R_{IB}(\phi, \theta, \psi)$ is the transformation matrix from the body frame(B) to the inertial frame(I), where $\phi, \theta, \psi$ are Euler angles.

With the on-board controller for the attitude stabilization, the unknown parameters in the equation (2). Since no exact description for the attitude controller exists and the control parameters are unknown, the total attitude controller’s dynamics including the rotor and blade dynamics are to be identified in the modeling task. For this purpose a linear black box modeling approach is used [7].

![Fig. 3. Black box model of the quadrotor’s attitude controller](image)

As illustrated in Fig 3, the four decoupled subsystems used to model the roll, pitch, yaw and thrust dynamics are approximated with a first order system.
2.3 Parameter Identification

The prediction error method is adopted as the identification algorithm. The features of system identification are symbolically indicated in Fig. 4.

![Fig. 4. The overall identification procedure](image)

The dynamic model being defined as linear time invariant, the relationship between input and output is as follows:

\[
y(t) = \hat{G}(q, \tau)u(t) + \hat{H}(q, \tau)e(t)
\]  

(3)

where \( e(t) \) is the white noise (unpredictable), \( \hat{H} \) is characteristics of noise. \( \hat{G} \) is dynamic model of system, its input is \( u(t) \), and predicted output is \( \hat{y}(t) \). In the black-box model approach, \( \hat{G} \) and \( \hat{H} \) are rational transfer functions in the shift (delay) operator \( q \) and unknown parameter \( \tau \). The prediction error method uses an iterative nonlinear least squares algorithm to minimize a cost function, which consists of a weighted sum of the square of the errors. The cost function is defined as follows:

\[
V_N(\hat{G}, \hat{H}) = \sum_{t=1}^{N} \varepsilon^2(t)
\]  

(4)

where \( \varepsilon \) denotes the error between the estimated and the measured values. For linear systems the error can be expressed as:

\[
\varepsilon(t) = y(t) - \hat{y}(t) = \hat{H}(q, \theta)^{-1}(y(t) - \hat{G}(q, \theta)u(t))
\]  

(5)

\( \varepsilon \) is equal the white noise \( e \) if the output was indeed generated by model and for some sequences \( u(t) \) and \( y(t) \). To increase the accuracy of the estimation, the algorithm was applied several times, using the prior estimation as the new initial guess. Through the above mentioned process of identification, below Table 1 presents predicted models from each experiment. The units of command of all transfer functions are in digital value (0~4095), which is obtained from RC transmitter, and the outputs are in degree or g unit.

<table>
<thead>
<tr>
<th>Decoupled Subsystem</th>
<th>Dynamic Model (Transfer function)</th>
</tr>
</thead>
</table>
| Roll                | \[
\phi = \frac{0.0142}{0.0945s + 1}
\] |
| Pitch               | \[
\theta = \frac{0.0138}{0.1034s + 1}
\] |
3 Dynamic Model Validation

We performed experiments four times near hover condition for the model validation, i.e. comparing the flight data with the response of the model simulated with the corresponding input data. The results of validation are presented in the Table 2 with the fitting rate, which is the percentage of output variance between simulated and measured output. The Fig. 5 shows the results of the system validation. For a comparison between the simulated thrust values and the thrust values measured by the IMU (Inertia Measurement Unit) a coordinate transformation from the body frame into the inertia frame was required. As can be seen in the plots of the validation, the dynamical model approximates the behavior of the real plant of the quadrotor quite well.

Table 2. Results of Fitting rate

<table>
<thead>
<tr>
<th>Thrust Model [%]</th>
<th>Roll Model [%]</th>
<th>Pitch Model [%]</th>
<th>Yaw Model [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test #1</td>
<td>68.8</td>
<td>84.23</td>
<td>88.76</td>
</tr>
<tr>
<td>Test #2</td>
<td>68.69</td>
<td>83.72</td>
<td>88.24</td>
</tr>
<tr>
<td>Test #3</td>
<td>67.7</td>
<td>82.88</td>
<td>88.68</td>
</tr>
<tr>
<td>Test #4</td>
<td>68.48</td>
<td>83.52</td>
<td>88.74</td>
</tr>
</tbody>
</table>

Fig. 5. The original response and estimated response of the decoupled models
4 Conclusions

In this paper, a practical system identification method is applied to obtain a linear time-invariant (LTI) model of a quadrotor using input-output flight data as opposed to painstaking mathematical modeling techniques. The system response data from Ascending Technologies Hummingbird quadrotor is acquired in carefully devised experiment procedure and then an LTI model is obtained using the prediction error method with the black-box model approach. The fidelity verifications show that the proposed method provides a fairly accurate model of the quadrotor flying robot. This acquisition of high fidelity system model of the quadrotor will be a crucial step for the successful design of high-performance flight control system.

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References