Advanced Science and Technology Letters
Vol.111 (NGCIT 2015), pp.35-41
http://dx.doi.org/10.14257/astl.2015.111.08

Two-sided Matching Decision under Multi-granularity Uncertain Linguistic Environment

[1a]Qi Yue, [2]Yongshan Peng, [3]Bingwen Yu, [4]Yu Hong*, [1b]Quan Xiao

[1]School of Information Management, Jiangxi University of Finance and Economics, Nanchang, China
[1a]yueqichina@126.com, [1b]xiaoquan@foxmail.com
[2]School of Physical Education, Jiangxi University of Finance and Economics, Nanchang 330013, China
pengyongshanjxcd@sina.com
[3]School of Economics, Jiangxi University of Finance and Economics, Nanchang 330013, China
yubingwen2005@163.com
[4]College of Humanities and Sciences, Northeast Normal University, Changchun 130117, China
*acjlhong@126.com

Abstract. With respect to the two-sided matching problem, where the agents' preferences are in the format of multi-granularity uncertain linguistic terms, a matching method is proposed. Some basic concepts and definitions on uncertain linguistic term and 2-tuple are firstly given. Based on this, the two-sided matching problem with multi-granularity uncertain linguistic terms is described. Then, multi-granularity uncertain linguistic term matrixes are transformed into multi-granularity 2-tuple matrixes. Furthermore, a multi-objective optimization model is developed by using the extended 2-tuple weighted average. By using the normalization method and the linear weighted method twice, the multi-objective optimization model can be converted into a single-objective optimization model. By solving the model, the matching alternative can be obtained. Example of matching demand and supply in agricultural knowledge illustrates the feasibility and effectiveness of this method.

Keywords: two-sided matching; multi-granularity; uncertain linguistic term; 2-tuple; optimization model.

1 Introduction

The two-sided matching problems widely exist in the real world. Examples contain the stable marriage problem [1-4], the CEOs selection problem [5-8], the college admission problem [9-12], the employee selection problem [13-15], and the personnel assignment problem [16-18]. Since the reasonable and effective matching alternative has a promoting effect for improving the efficiency of economic management activities and the satisfaction degrees of two-sided agents, studying on the two-sided matching problems has important theoretical significance and practical application value.

ISSN: 2287-1233 ASTL
Copyright © 2015 SERSC

There are plenty of two-sided matching problems in many fields of real life, such as marriage assignment [1], college admission [2], employee selection [3], personnel assignment [4] and CEOs to companies [5], Therefore two-sided matching is a research topic with extensive application backgrounds.

The remainder of this paper is arranged as follows: Section 2 introduces some concepts of uncertain linguistic term and 2-tuple. Section 3 formulates the two-sided matching problem with multi-granularity uncertain linguistic terms. Section 4 proposes a new method. Section 5 gives a matching example in agricultural knowledge. Section 6 summarizes the main features of the proposed method.

2 The Problem

This paper considers the two-sided matching problem, where the agents' preferences are in the format of multi-granularity uncertain linguistic terms. Let $A=\{A_1,A_2,\ldots,A_m\}$ $(m\geq 2)$ be the set of agents of side A, where A_i denotes the i th agent of side A; $B=\{B_1,B_2,\ldots,B_n\}$ $(n\geq m\geq 2)$ be the set of agents of side B, where B_j denotes the j th agent of side B. Let $S_{A_i}=\{s_0,s_1,\cdots,s_{p_i}\}$ be the set of satisfaction linguistic terms given by A_i, where p_i+1 denotes the cardinality of set S_{A_i}; $S_{B_j}=\{s_0,s_1,\cdots,s_{q_j}\}$ be the set of satisfaction linguistic terms given by B_j, where q_j+1 denotes the cardinality of set S_{B_j}. Let $\tilde{L}_A=[\tilde{l}_{ij}^A]_{m\times n}$ be the multi-granularity uncertain linguistic term matrix from side A to B, where $\tilde{l}_{ij}^A=[l_{ij}^{AL},l_{ij}^{AU}]$ denotes the multi-granularity uncertain linguistic term for agent A_i towards B_j, $l_{ij}^{AL},l_{ij}^{AU}\in S_{A_i}$; $\tilde{L}_B=[\tilde{l}_{ij}^B]_{m\times n}$ be the multi-granularity uncertain linguistic term matrix from side B to A, where $\tilde{l}_{ij}^B=[l_{ij}^{BL},l_{ij}^{BU}]$ denotes the multi-granularity uncertain linguistic term for agent B_j towards A_i, $l_{ij}^{BL},l_{ij}^{BU}\in S_{B_j}$. Let μ be the two-sided matching or matching alternative, $\mu=\mu_T\cup\mu_O$. Here, μ_T denotes the set of matching pairs, $\mu_T=\{(A_i,B_{\sigma(i)})\,|\,i=1,\cdots,m\}$, where $\sigma(1),\cdots,\sigma(m)$ is the permutation of $1,2,\cdots,m$; μ_O denotes the set of single pairs, $\mu_O=\{(B_j,B_j)\,|\,j\in\{1,\cdots,n\}\setminus\{\sigma(1),\cdots,\sigma(m)\}\}$.

The considered problem is how to obtain the reasonable matching alternative based on multi-granularity uncertain linguistic term matrixes $\tilde{L}_A=[\tilde{l}_{ij}^A]_{m\times n}$ and $\tilde{L}_B=[\tilde{l}_{ij}^B]_{m\times n}$.

 Copyright © 2015 SERSC

3 The Method

This section will present a matching method for solving the two-sided matching problem with multi-granularity uncertain linguistic terms.

Firstly, by Eqs. (1a)-(3), we transform multi-granularity uncertain linguistic term matrixes $\tilde{L}_A = [\tilde{l}_{ij}^A]_{m \times n}$ and $\tilde{L}_B = [\tilde{l}_{ij}^B]_{m \times n}$ into multi-granularity 2-tuple matrixes $L_A = [(l_{ij}^A, \alpha_{ij})]_{m \times n}$ and $L_B = [(l_{ij}^B, \beta_{ij})]_{m \times n}$, where multi-granularity 2-tuples (l_{ij}^A, α_{ij}) and (l_{ij}^B, β_{ij}) are calculated by

$$(l_{ij}^A, \alpha_{ij}) = \theta\left(\frac{\theta^{-1}(l_{ij}^{AL}, 0) + \theta^{-1}(l_{ij}^{AU}, 0)}{2}\right) \tag{5}$$

$$(l_{ij}^B, \beta_{ij}) = \theta\left(\frac{\theta^{-1}(l_{ij}^{BL}, 0) + \theta^{-1}(l_{ij}^{BU}, 0)}{2}\right) \tag{6}$$

Based on multi-granularity 2-tuple matrixes L_A and L_B, we consider to construct an optimization model for obtaining the matching alternative considering matching constraints. On the one hand, according to the characteristics of linguistic term and 2-tuple, we know that the greater l_{ij}^A or l_{ij}^B is, the greater satisfaction degree of agent A_i or B_j is. Hence, we take the maximization 2-tuple as the objective function. On the other hand, due to $m \le n$, the matching constraints can be interpreted as follows: agent A_i has exactly one matching agent of side B (i.e., $\sum_{j=1}^{n} x_{ij} = 1$), and agent B_j has at most one matching agent of side A (i.e., $\sum_{i=1}^{m} x_{ij} \le 1$), where $x_{ij} = \begin{cases} 0, & \mu(A_i) \ne B_j \\ 1, & \mu(A_i) = B_j \end{cases}$. Moreover, by using the extended 2-tuple weighted average, the following multi-objective optimization model (7) can be set up:

$$\max \quad Z(A_i) = \theta\left(\sum_{j=1}^{n} \theta^{-1}(l_{ij}^A, \alpha_{ij}) x_{ij}\right), i = 1, 2, \ldots, m \tag{7a}$$

$$\max \quad Z(B_j) = \theta\left(\sum_{i=1}^{m} \theta^{-1}(l_{ij}^B, \beta_{ij}) x_{ij}\right), j = 1, 2, \ldots, n \tag{7b}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} = 1, i = 1, 2, \ldots, m \tag{7c}$$

Copyright © 2015 SERSC

$$\sum_{i=1}^{m} x_{ij} \le 1,\ j = 1,2,\ldots,n \tag{7d}$$

$$x_{ij} = 0 \text{ or } 1,\ i = 1,2,\ldots,m,\ j = 1,2,\ldots,n \tag{7e}$$

In order to solve model (7), considering that symbolic translation θ is a monotone increasing function, the optimal solution(s) of the following optimization model (8) is that of model (7):

$$\max \quad Z(A_i) = \sum_{j=1}^{n} \theta^{-1}(l_{ij}^{A}, \alpha_{ij}) x_{ij}, i = 1,2,\ldots,m \tag{8a}$$

$$\max \quad Z(B_j) = \sum_{i=1}^{m} \theta^{-1}(l_{ij}^{B}, \beta_{ij}) x_{ij}, j = 1,2,\ldots,n \tag{8b}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} = 1, i = 1,2,\ldots,m \tag{8c}$$

$$\sum_{i=1}^{m} x_{ij} \le 1,\ j = 1,2,\ldots,n \tag{8d}$$

$$x_{ij} = 0 \text{ or } 1,\ i = 1,2,\ldots,m,\ j = 1,2,\ldots,n \tag{8e}$$

In order to solve model (8), considering that the cardinalities of sets $S_{A_i}\ (i = 1,2,\ldots,m)$ may be not equal, we should normalize Eqs. (8a) and (8b) firstly. Usually, the status of each agent of one side is the same. Hence, we can suppose that each agent of one side has equal priority. Then, using the linear weighted method, model (8) can be transformed into the following bi-objective optimization model (9):

$$\max \quad Z(A) = \frac{1}{m} \sum_{i=1}^{m} \frac{\sum_{j=1}^{n} \theta^{-1}(l_{ij}^{A}, \alpha_{ij}) x_{ij}}{p_i + 1} \tag{9a}$$

$$\max \quad Z(B) = \frac{1}{n} \sum_{j=1}^{n} \frac{\sum_{i=1}^{m} \theta^{-1}(l_{ij}^{B}, \beta_{ij}) x_{ij}}{q_j + 1} \tag{9b}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} = 1, i = 1,2,\ldots,m \tag{9c}$$

$$\sum_{i=1}^{m} x_{ij} \le 1,\ j = 1,2,\ldots,n \tag{9d}$$

$$x_{ij} = 0 \text{ or } 1,\ i = 1,2,\ldots,m,\ j = 1,2,\ldots,n \tag{9e}$$

Copyright © 2015 SERSC

In order to solve model (9), the linear weighted method is also used. Let w_A and w_B be the weight of objectives functions Z(A) and Z(B), respectively, such that $w_A, w_B \in [0,1]$, $w_A + w_B = 1$, then model (9) can be transformed into the following single-objective optimization model (10):

$$\max \quad Z = \left(\frac{w_A}{m} \sum_{i=1}^{m} \frac{\sum_{j=1}^{n} \theta^{-1}(l_{ij}^A, \alpha_{ij})}{p_i + 1} + \frac{w_B}{n} \sum_{j=1}^{n} \frac{\sum_{i=1}^{m} \theta^{-1}(l_{ij}^B, \beta_{ij})}{q_j + 1} \right) x_{ij} \tag{10a}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} = 1, i = 1, 2, \ldots, m \tag{10b}$$

$$\sum_{i=1}^{m} x_{ij} \le 1, j = 1, 2, \ldots, n \tag{10c}$$

$$x_{ij} = 0 \text{ or } 1,\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n \tag{10d}$$

Model (10) can be solved by the existing mathematical optimization software. According to multiple-objective programming theory, the optimal solution of model (10) is the efficient solution of model (9). Then the matching alternative can be obtained based on the obtained optimal solution.
In sum, an algorithm is developed to solve the two-sided matching problem with multi-granularity uncertain linguistic terms as follows:

Step 1. Transform multi-granularity uncertain linguistic term matrixes $\tilde{L}_A = [\tilde{l}_{ij}^A]_{m\times n}$ and $\tilde{L}_B = [\tilde{l}_{ij}^B]_{m\times n}$ into multi-granularity 2-tuple matrixes $L_A = [(l_{ij}^A, \alpha_{ij})]_{m\times n}$ and $L_B = [(l_{ij}^B, \beta_{ij})]_{m\times n}$ by Eqs (5) and (6).

Step 2. Built the multiple-objective optimization model (7) based on multi-granularity 2-tuple matrixes L_A and L_B by Eq. (4).

Step 3. Transform model (7) into model (8).
Step 4. Transform model (8) into model (9) by using the normalization method and the linear weighted method.
Step 5. Transform model (9) into model (10) by using the linear weighted method.
Step 6. Obtain the matching alternative by solving model (10).

4 The Example

The agricultural innovation intermediary in high-tech zone of Nanchang plans to make agricultural knowledge matchings between end-users of agricultural entrepreneurs (demand side) and producers of agricultural R&D and KIBS (supply

Copyright © 2015 SERSC

side). Four agricultural entrepreneurs A_1, A_2, …, A_4 and six producers B_1, B_2, …, B_6 participate in the process of matching. Agricultural entrepreneur A_i evaluates producers from market prospect, complexity, and price. Producer B_j evaluates agricultural entrepreneurs from income, conversion speed, and technological level. The satisfaction linguistic term sets $S_{A_1} = S_{A_2}$ ={ s_0 =VL(Very low), s_1 =L(Low), s_2 =ML(Medium low), s_3 =M(Medium), s_4 =MH(Medium high), s_5 =H(High), s_6 =VH(Very high)}, $S_{A_3} = S_{A_4}$ ={ s_0 =VL(Very low), s_1 =L(Low), s_2 =M(Medium), s_3 =H(High), s_4 =VH(Very high)}. The satisfaction linguistic term set $S_{B_1} = S_{B_2} = S_{B_3} = S_{B_4}$ ={ s_0 =VL(Very low), s_1 =L(Low), s_2 =M(Medium), s_3 =H(High), s_4 =VH(Very high)}, $S_{B_5} = S_{B_6}$ ={ s_0 =L(low), s_1 =M(Medium), s_2 =H(High)}. The multi-granularity uncertain linguistic term matrixes $\tilde{L}_A = [\tilde{l}_{ij}^A]_{4\times 6}$ and $\tilde{L}_B = [\tilde{l}_{ij}^B]_{4\times 6}$ are provided as follows.

$$\tilde{L}_A = \begin{bmatrix} [s_2,s_3] & [s_3,s_4] & [s_2,s_2] & [s_4,s_5] & [s_1,s_2] & [s_5,s_6] \\ [s_3,s_4] & [s_2,s_3] & [s_1,s_1] & [s_0,s_1] & [s_4,s_4] & [s_2,s_2] \\ [s_2,s_2] & [s_0,s_1] & [s_1,s_2] & [s_3,s_4] & [s_2,s_3] & [s_4,s_4] \\ [s_1,s_2] & [s_4,s_4] & [s_3,s_4] & [s_0,s_1] & [s_2,s_3] & [s_2,s_2] \end{bmatrix}$$

$$\tilde{L}_B = \begin{bmatrix} [s_2,s_3] & [s_3,s_4] & [s_2,s_2] & [s_3,s_3] & [s_1,s_1] & [s_0,s_1] \\ [s_3,s_4] & [s_2,s_3] & [s_1,s_1] & [s_2,s_3] & [s_1,s_2] & [s_2,s_2] \\ [s_2,s_2] & [s_0,s_1] & [s_2,s_3] & [s_1,s_2] & [s_0,s_1] & [s_1,s_2] \\ [s_1,s_2] & [s_2,s_2] & [s_3,s_4] & [s_4,s_4] & [s_2,s_2] & [s_1,s_2] \end{bmatrix}$$

To determine the reasonable matching alternative, a brief description of the matching process is given below. Firstly, by Eqs. (5) and (6), multi-granularity uncertain linguistic term matrixes $\tilde{L}_A = [\tilde{l}_{ij}^A]_{4\times 6}$ and $\tilde{L}_B = [\tilde{l}_{ij}^B]_{4\times 6}$ are transformed into multi-granularity 2-tuple matrixes $L_A = [(l_{ij}^A, \alpha_{ij})]_{4\times 6}$ and $L_B = [(l_{ij}^B, \beta_{ij})]_{4\times 6}$. Then, the multiple-objective optimization model (7) can be built by Eq. (4). By using the normalization method and the linear weighted method, model (7) can be transformed into model (9). Suppose $w_A = 0.6$ and $w_B = 0.4$, model (9) can be transformed into model (10), where coefficient matrix $C = (c_{ij})_{4\times 6}$ is calculated by

 Copyright © 2015 SERSC

$$C = \begin{bmatrix} 0.0869 & 0.1217 & 0.0695 & 0.1364 & 0.0544 & 0.129 \\ 0.1217 & 0.0869 & 0.0348 & 0.044 & 0.119 & 0.0873 \\ 0.0867 & 0.0217 & 0.0783 & 0.125 & 0.0861 & 0.1533 \\ 0.065 & 0.1467 & 0.145 & 0.0683 & 0.1194 & 0.0933 \end{bmatrix}$$

Lastly, by solving model (10), the matching alternative $\mu = \mu_T^* \cup \mu_O^*$ can be obtained, i.e.,

$$X^* = [x_{ij}^*]_{4\times 6} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

According to the unique optimal solution X^*, the matching alternative μ^* can be obtained, i.e., $\mu^* = \mu_T^* \cup \mu_O^*$, where $\mu_T^* = \{(A_1, B_4), (A_2, B_1), (A_3, B_6), (A_4, B_2)\}$, $\mu_O^* = \{(B_3, B_3), (B_5, B_5)\}$. Hence, entrepreneur A_1 matches with producer B_4, entrepreneur A_2 matches with producer B_1, entrepreneur A_3 matches with producer B_6, entrepreneur A_4 matches with producer B_2, producers B_3 and B_5 are single.

References

1. R. W. Irving, D. F. Manlove, and G. O'Malley, Journal of Discrete Algorithms, 7, 213, (2009).
2. M. Clark, J. Rothstein, and D. W. Schanzenbach, Econ. Educ. Rev. 28, 295, (2009).
3. K. J. Lauver, and A. Kristof-Brown, J. Vocat. Behav. 59, 454, (2001).
4. A. E. Roth, and E. Peranson, Am. Econ. Rev. 89, 748, (1999).
5. X. Gabaix, and A. Landier, Q. J. Econ. 123, 49 (2008).
6. D. Gale, and L. Shapley, Am. Math. Mon. 69, 9 (1962).
7. A. Mauleon, V. J. Vannetelbosch, and W. Vergote, Theoretical Economics, 6, 499, (2011).
8. B. H. Boon, and G. Sierksma, Eur. J. Oper. Res. 148, 277 (2003).
9. J. Sethuraman, C. P. Teo, and L. Qian, Math. Oper. Res. 31, 581, (2006).
10. L. Ehlers, Math. Oper. Res. 33, 327 (2008).
11. K. Uetake, Y. Watanabe, Econ. Lett. 116, 535, (2012).
12. F. Herrera, and E. Herrera-Viedma, Syst. Man Cy. A. 27, 646, (1997).
13. F. Herrera, E. Herrera-Viedma, and L. Martinez, Fuzzy Set. Syst. 114, 43, (2000).

Copyright © 2015 SERSC