

## Design of Optical Lens for Quasi-diffraction-free beam using Particle Swarm Optimization

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**Abstract.** Quasi-diffraction-free beams are produced by an annular aperture placed in the back focal plane of a thin lens. Since the width of the annular aperture is narrow, the energy efficiency of the beam is very low. In this paper, the device for generating the quasi-diffraction-free beams using an axicon lens with a corn-shaped notch is proposed to improve its energy efficiency. The geometrical parameters of the axicon lens are determined by the optimization algorithm (Particle Swarm Optimization). The numerically calculated results are shown.

**Keywords:** Optimization algorithm, Particle Swarm Optimization (PSO), Quasi-diffraction-free beam, Axicon lens.

### 1 Introduction

Diffraction-free beams (DFBs) were first discovered by Durnin [1]. The DFB-generation device proposed by Durnin consists of a thin lens and an annular aperture with infinitely small width. The annular aperture is placed in the back focal plane of the thin lens. Theoretically, the DFB propagates without attenuation to infinite region. However, the DFB must have infinitely big institute to transmit through the annular aperture. Therefore, it is difficult to generate the DFB using the device proposed by Durnin.

Tanaka et al. investigated the DFB generated by the annular aperture with the finite width [2]. It was shown that the propagated distance of the DFB depends on the ratio of the outer radius to the inner radius of the annular aperture and is finite region by numerical calculation. The DFB investigated by Tanaka et al. was referred to as a quasi-diffraction-free beam (QDFB). Although the DFB can be generated by the aperture with the finite width, the width is narrow (the ratio of the inner radius to the outer radius is more than 0.9). Therefore, most incident waves are interrupted by the annular aperture. The energy efficiency of the QDFB is very low.

In this paper, the device for generating the QDFB with high energy efficiency is proposed. The QDFB with high energy efficiency is suitable for optical space communication and optical power transmission. An axicon lens with a corn-shaped notch is placed in front of the annular aperture to improve the energy efficiency. In general, it is difficult to design the optical lens satisfies the required specifications because the wavelength of the optics is very small compared with the optical lens. Therefore, difference in slight size of the lens influences its characteristics. In this paper, the geometrical parameters of the axicon lens are determined by Particle Swarm Optimization (PSO) [3]. PSO is a relatively simple optimization algorithm. This algorithm is based on the stochastic optimization technique and is inspired by social behavior of bird flocking or fish schooling. PSO is applied to the design of the axicon lens to obtain the intensity of the wave transmitted from the axicon lens as great as possible.

## 2 Theory

Figure 1 shows the geometrical configuration of the QDFB-generation device. A conventional QDFB-generation device consists of an annular aperture and a thin lens of a focal length  $f$ . The distance between the annular aperture and the thin lens is  $f$ . In the proposed QDFB-generation device, an axicon lens with a corn-shaped notch is installed in front of the annular aperture. The QDFB is produced behind the thin lens. The outer and inner radii of the annular aperture are  $a_1$  and  $b_1$ , respectively. The radius of the thin lens is  $a_2$ . The radii of the axicon lens and the notch are  $a_0$  and  $b_0$ , respectively. The base angles of the axicon lens and the notch are  $\alpha_2$  and  $\alpha_1$ , respectively.

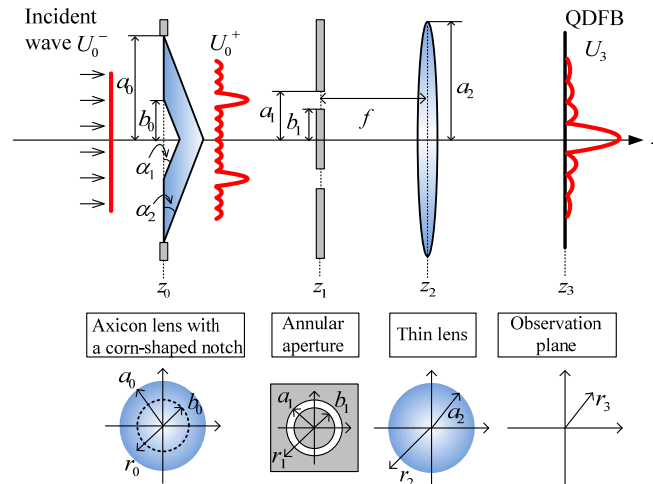


Fig. 1. Geometrical configuration of the QDFB-generation device.

Fresnel diffraction pattern  $U_0^+$  transmitted through the axicon lens is expressed as

$$U_0^+ = j \frac{F}{Z_a} \exp\left(-j \frac{K^2}{F} Z_a - j \frac{FR_1^2}{2Z_a}\right) \int_0^{A_0} U_0^- T_0(R_0, Z_a = 0) \exp\left(-j \frac{FR_0^2}{2Z_a}\right) J_0\left(\frac{FR_0 R_1}{Z_a}\right) R_0 dR_0. \quad (1)$$

where  $U_0^-$  is the incident wave into the axicon lens with a corn-shaped notch and  $J_0$  is Bessel function of order 0. The transmittance coefficient of the axicon lens  $T_0$  is given by

$$T_0 = \begin{cases} \exp\{-jK(n-1)\} \left[ (1-R_0) \tan \alpha_2 - (B_0 - R_0) \tan \alpha_1 \right] & : 0 \leq R_0 \leq B_0 \\ \exp\{-jK(n-1)\} (1-R_0) \tan \alpha_2 & : B_0 \leq R_0 \leq 1 \end{cases} \quad (2)$$

$n$  is the refractive index of the axicon lens. Fresnel diffraction pattern  $U_3$  behind the thin lens, the proposed QDFB, is expressed as

$$U_3 = -j \frac{F^3}{Z_a Z_c} \exp\left\{-j \frac{K^2}{F} (Z_a + 1 + Z_c) - j \frac{FR_3^2}{2Z_c}\right\} \\ \times \int_0^{A_2} \int_{\varepsilon A_1}^{A_1} \int_0^{A_0} U_0^- T_0(R_0, Z_a = 0) \exp\left[-j \frac{F}{2} \left(\frac{R_0^2}{Z_a} + R_1^2 + \frac{R_2^2}{Z_c}\right)\right] \\ \times J_0\left(\frac{FR_0 R_1}{Z_a}\right) J_0(FR_1 R_2) J_0\left(\frac{FR_2 R_3}{Z_c}\right) R_0 R_1 R_2 dR_0 dR_1 dR_2, \quad (3)$$

where, the non-dimensional parameters in Eqs. (1) - (3) are defined as follows.

$$R_0 = \frac{r_0}{a_0}, R_1 = \frac{r_1}{a_0}, R_2 = \frac{r_2}{a_0}, R_3 = \frac{r_3}{a_0}, Z_a = \frac{z_1 - z_0}{f}, Z_c = \frac{z_3 - z_2}{f}, \\ A_0 = 1, B_0 = \frac{b_0}{a_0}, A_1 = \frac{a_1}{a_0}, A_2 = \frac{a_2}{a_0}, \varepsilon = \frac{b_1}{a_1}, K = ka_0, F = \frac{ka_0^2}{f} \quad (4)$$

Since the effects of the finite aperture of the thin lens have already been discussed [1][4][5], the radius of the aperture of the thin lens is set to be infinite. By applying the formulas [6]

$$\int_0^\infty x \exp(-a^2 x^2) J_m(px) J_m(qx) dx = \frac{1}{2a^2} \exp\left(-\frac{p^2 + q^2}{4a^2}\right) J_m\left(\frac{pq}{2a^2}\right) \quad (5)$$

to Eq. (3), the proposed QDFB is reduced to the following equation.

$$U_3 = -\frac{F^2}{Z_a} \exp\left\{-j \frac{K^2}{F} (Z_a + 1 + Z_c) + jF \frac{R_1^2 Z_c}{2}\right\} \\ \times \int_{\varepsilon A_1}^{A_1} \int_0^{A_0} U_0^- T_0(R_0, Z_a = 0) \exp\left[-jF \left(\frac{R_0^2}{2Z_a} + R_1^2 + \frac{R_1^2}{2Z_a}\right)\right] \\ \times J_0\left(\frac{FR_0 R_1}{Z_a}\right) J_0(FR_1 R_3) R_0 R_1 dR_0 dR_1. \quad (6)$$

In order to improve energy efficiency of the QDFB, the following conditions are required to the diffraction wave transmitted through the axicon lens.

- (i) the diffraction wave behind the axicon lens distributes annularly.
- (ii) the intensity of the diffraction wave is as big as possible.

In this study, the geometrical parameters of the axicon lens are determined by applying PSO to the axicon lens. The geometrical parameters decided by PSO are  $B_0$ ,  $\alpha_1$ ,  $\alpha_2$ , and  $Z_a$ . The fitness function is given by the following equation.

$$|U_0^+(R_1, Z_a)|^2 \Rightarrow \text{Maximum} \quad \text{and} \quad |U_0^+(R_1 = 0, Z_a)|^2 \leq 5 \quad (7)$$

where  $U_0^+$  is the transmitted wave from the axicon lens. In the geometry of the axicon lens with a corn-shaped notch, additional conditions as shown below are imposed

$$0.1 \leq B_0 \leq 0.9, \quad \alpha_1, \alpha_2 \geq 0.5^\circ, \quad Z_a \leq 0.2 \quad (8)$$

### 3 Results and Discussion

Table 1 shows the values of the geometrical parameters optimized by PSO. The intensity of the incident wave  $U_0^-$  into the axicon lens is unity. The parameters  $F$  and  $K$  are fixed to 20 and 10000 in Eq. (1), respectively. In PSO, the numbers of the population and the generation are 20 and 300, respectively.

**Table 1.** Values of the geometrical parameters of the axicon lens optimized by PSO.

$B_0$	$\alpha_1$	$\alpha_2$	$Z_a$
0.58835	0.50832	0.92133	0.11887

Figure 2 shows the calculated distribution of the diffraction pattern  $|U_0^+|^2$  behind the axicon lens. The maximum intensity is 232.14 at  $R_1=0.20$ , that is, it is confirmed that the transmitted wave  $U_0^+$  with the intensity =232.14 is produced annularly behind the axicon lens.

### 4 Conclusion

The optical axicon lens with a corn-shaped notch is designed by the optimization algorithm PSO. The diffraction pattern with annular distribution is obtained behind the axicon lens and its intensity is approximately 232 times greater than the incident wave (plane wave). It was confirmed that PSO was useful for the design of the optical lens by numerical calculation. It is expected that the institute of the QDFB proposed in this paper is also approximately 232 times greater than the conventional QDFB. The future works is to elucidate the characteristics of the QDFB proposed in this paper.

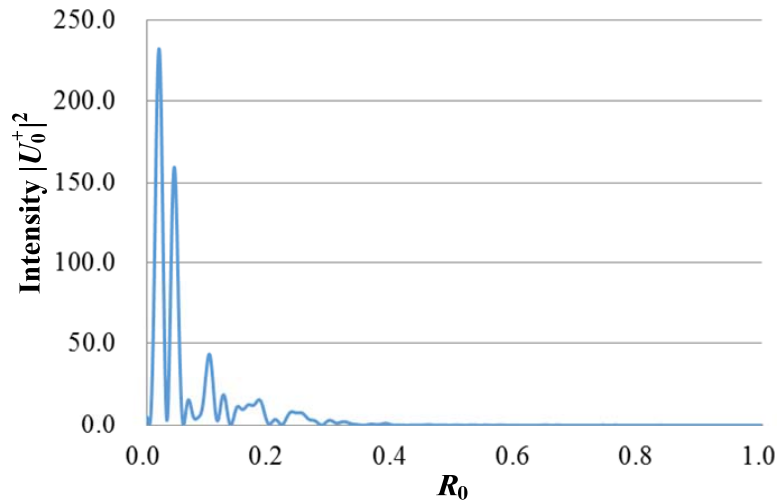


Fig. 2. Radial distribution of the wave transmitted from the axicon lens.

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